MATLAB Assignment #4

Linear Systems, Straight Line Solutions, and Matrix Exponentials

See MATLAB files *ExampleLinearSystems.m* and *ExampleMatrix.m* for a helpful start on completing this assignment. You can use MATLAB to do your matrix algebra for you, such as computing eigenvalues and eigenvectors. All of the matrices in this assignment have eigenvalues which are integers, and their eigenvectors can be written with integer components.

1 Consider the matrix $A = \begin{pmatrix} -3 & -5 \\ 2 & 4 \end{pmatrix}$.

(a) The matrix exponential e^{At} can be written as the power series $\sum_{k=0}^{\infty} \frac{1}{k!}(At)^k$. Calculate the partial sums $S_M = \sum_{k=0}^{M} \frac{1}{k!}(2A)^k$ for $M = 1, 2, 3, 4$. Your answers should have at least one decimal point of precision.

(b) Find a matrix T and a diagonal matrix $L = \left(\begin{smallmatrix} a & 0 \\ 0 & b \end{smallmatrix}\right)$ such that $A = TLT^{-1}$.

(c) Use the formula $e^{At} = Te^{Lt}T^{-1}$ to calculate e^{2A}.

(d) Use the MATLAB function *expm* to calculate e^{2A}. How does it compare with your answers to part (a) and (c)? Does the series in part (a) seem to converge quickly or slowly?

2) For the matrix $B = \begin{pmatrix} 3 & 5 \\ -5 & -7 \end{pmatrix}$ we study the linear differential equation: $\frac{d\mathbf{Y}}{dt} = B \cdot \mathbf{Y}$. This matrix A has one eigenvector $V_0 = (1, -1)$ and its corresponding eigenvalue is $\lambda_0 = -2$.

(a) Write down a solution $\mathbf{Y}(t)$ to the initial value problem $\mathbf{Y}(0) = (-3, 3)$.

(b) Plot a direction field for the linear differential equation $\frac{d\mathbf{Y}}{dt} = B \cdot \mathbf{Y}$.

(c) On top of your vector field, plot your the solution $Y(t)$ from part *(b)* for the period of time $0 \le t \le 5$.

3) For the matrix $C = \begin{pmatrix} -3 & 1 \\ 1 & -3 \end{pmatrix}$ we study the linear differential equation: $\frac{d\mathbf{Y}}{dt} = C \cdot \mathbf{Y}$.

(a) Calculate the eigenvalues and corresponding eigenvectors of the matrix C. This matrix C has two distinct, real eigenvalues. Write your eigenvectors V_1 and V_2 so that each component of the vectors is an integer.

(b) The linearity principal says that if Y_1 and Y_2 are solutions then $Y_3(t) = Y_1(t) + Y_2(t)$ is also a solution. Find two straight line solutions $Y_1(t)$ and $Y_2(t)$ such that $Y_3(t) := Y_1(t)+Y_2(t)$ is a solution with initial conditions $Y_3(0) = (4, 10)$.

(c) For this question you will produce three figures. Each figure should have a direction field of the differential equation $\frac{d\mathbf{Y}}{dt} = C \cdot \mathbf{Y}$. On top of this direction field plot the three solutions $Y_1(t)$, $Y_2(t)$, $Y_3(t)$ from part (b) for time $0 \le t \le t_{max}$. Do this for values $t_{max} = 0.5, 1, 2$.