Name: ______________________________

MATLAB Assignment #3

Modeling and the Geometry of Systems

1. For this problem, we study the nonlinear differential equation:

$$\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= x - x^3 - y^3 \end{aligned}$$

a) Algebraically determine all of the equilibria to the differential equation

b) Run the MATLAB file *ExampleDirectionField.m* to generate a direction field of the differential equation. You might want to change the default spacing between the arrows in the direction field. Include this image in your submission.

c) Using the direction field from part b), draw by hand the phase portrait for the differential equation.

d) For a solution $\{x(t), y(t)\}$ with $\{x(0), y(0)\} = \{x_0, y_0\}$, use your phase diagram to describe the long term behavior of the solution.

1. $\{x_0, y_0\} = \{1, 1\}$
2. $\{x_0, y_0\} = \{-1, -1\}$
3. $\{x_0, y_0\} = \{2, -1.5\}$
4. $\{x_0, y_0\} = \{1.5, -2\}$

2. A slinky is dangled from atop a high building, and its relative position is modeled by the second order differential equation

$$y'' + 2y' + y = 0$$

where y is the height of the bottom of the slinky.

a) Convert this second order differential equation into a 2-dimensional first order differential equation using the variable $v = y'$.

b) Modify the file *H.m* so that it matches your differential equation from part a). Then run the MATLAB file *ExampleDirectionField.m* to generate a direction field of the differential equation. Include this image in your submission.

c) Using your direction field from part b), draw by hand the phase portrait for the differential equation.

d) Write an interpretation of your graphs in terms of the typical behavior of a slinky.

e) Sketch by hand both the $y(t)$ and $v(t)$ graphs for the initial position $y(0) = -2$ and $v(0) = 0$.