GRE MATHEMATICS
TEST V

TIME: 2 hours and 50 minutes
66 Questions

DIRECTIONS: Choose the best answer for each question and mark the letter of your selection on the corresponding answer sheet.

1. $\lim_{n \to \infty} \left(\prod_{m=2}^{n} \left(1 - \frac{1}{m}\right) \right)$ is equal to

(A) 1
(B) e
(C) π
(D) e^{-1}
(E) 0

2. If $n = 2^{20}$, then what is the sum of all integer divisors d of n, $1 \le d \le n$?

(A) $2^{22} + 1$
(B) $2^{22} - 1$
(C) $2^{21} - 1$
(D) $2^{21} + 1$
(E) $2^{20} + 1$

3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and A^{-1} is the inverse of A, what is the determinant of A^{-1}?

(A) -2

(B) -5

(C) $\frac{1}{5}$

(D) $-\frac{1}{2}$

(E) 2

4. Which of the following complex numbers is equal to $(1+i)^{\frac{4}{3}}$?

(A) $2^{\frac{2}{3}}\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$

(B) $2^{\frac{2}{3}}\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$

(C) $2^{\frac{2}{3}}\left(\sqrt{\frac{3}{2}}+i\frac{1}{2}\right)$

(D) $2^{\frac{2}{3}}\left(\sqrt{\frac{3}{2}}-i\frac{1}{2}\right)$

(E) $2^{\frac{2}{3}}\left(\sqrt{\frac{2}{3}}+i\frac{1}{\sqrt{3}}\right)$

5. What is the value of $5\log_{15}(15x)-\log_{15}x^5$?

(A) 5

(B) $105x^2$

(C) 105

(D) $5\log_{15}\frac{15}{x}$

(E) $5\log_{15}4x$

6. Find the value of $\lim_{x\to 0}\frac{1}{x}\int_0^x (1+\sin t)^t\,dt$.

(A) 1

(B) $\frac{3}{2}$

(C) e

(D) $\frac{2}{3}$

(E) e^{-1}

7. What is the value of $\lim_{h\to 0}\frac{\sqrt{3+h}-\sqrt{3}}{h}$?

(A) 0

(B) $\frac{\sqrt{3}}{3}$

(C) Undefined

(D) $\frac{\sqrt{3}}{6}$

(E) ∞

8. How many solutions are there of the equation

$$\cos^2 x = \cos x \text{ if } 0 \le x \le 2\pi\,?$$

(A) No solutions

(B) 1

(C) 2

(D) 3

(E) 4

9. Given a set S of strictly negative real numbers, what is the greatest lower bound of the set $\{ x \text{ real} \mid x \text{ is an upper bound of } S \}$?

(A) 0

(D) $-\infty$

(B) Infimum of s

(E) Does not exist

(C) supremum of s

10. The partial derivative $\frac{\partial}{\partial y}\left[\int_0^1 e^{y \sin x}\, dx\right]$ is equal to

(A) $\int_0^1 e^{y \sin x}\, dx$

(D) $\int_0^1 e^{y \sin x} (\sin y)\, dy$

(B) $\int_0^1 \cos x\, e^{y \sin x}\, dx$

(E) $\int_0^1 y \sin x\, e^{y \sin x}\, dx$

(C) $\int_0^1 \sin x\, e^{y \sin x}\, dx$

11. If $f(x, y) = \frac{2}{x^2} + 3xy$ for $x \neq 0$, and the gradient of f at (r, s) has length r, then which of the following equations is satisfied by r and s ?

(A) $16 + 24 r^3 s + 9r^6 s^2 + 8 r^8 = 0$

(B) $24 + 9 r^3 s + r^6 s^2 + 8 r^8 = 0$

(C) $16 - r^3 s + 9r^6 s^2 + 8 r^8 = 0$

(D) $16 - 24 r^3 s + 9r^6 s^2 + r^8 = 0$

(E) $16 - 24 r^3 s + 9r^6 s^2 + 8 r^8 = 0$

12. Given $f(x) = \frac{x}{x-1}$, find an expression of $f(3x)$ in term of $f(x)$.

(A) $\frac{3f(x)}{3f(x) - 1}$

(D) $\frac{3f(x)}{2f(x) + 1}$

(B) $\frac{3f(x)}{3f(x) - 3}$

(E) $\frac{3f(x)}{2f(x) - 1}$

(C) $3f(x) - 1$

13. Let $S_n = -1 + 2\left(\frac{2}{3}\right) - 3\left(\frac{2}{3}\right)^2 + \ldots + (-1)^n n \left(\frac{2}{3}\right)^{n-1}$.

What is $\lim_{n \to \infty} S_n$?

(A) $-\frac{9}{25}$

(D) $-\frac{25}{9}$

(B) $\frac{9}{25}$

(E) $\frac{5}{3}$

(C) $\frac{25}{9}$

If the domain of function $y = f(x)$ is $[0, 1]$, then what is the domain of function $f(x + \frac{1}{4}) + f(x - \frac{1}{4})$?

(A) $[0, 1]$

(B) $(0, 1)$

(C) $[0, \frac{1}{2}]$

(D) $[\frac{1}{4}, \frac{3}{4}]$

(E) $(\frac{1}{4}, \frac{3}{4})$

$[\sqrt{2}(1 - i)]^{48} =$

(A) 2^{24}

(B) -2^{24}

(C) 2^{48}

(D) -2^{48}

(E) $2^{24}(1 - i)$

The solution of differential equation $y\,dx + \sqrt{x^2+1}\,dy = 0$ is:

(A) $y\left(x + \sqrt{x^2 + 1}\right) = c$

(B) $y\left(1 + \sqrt{x^2 + 1}\right) = c$

(C) $xy + \sqrt{x^2 + 1} = c$

(D) $yx + \frac{y}{\sqrt{x^2 + 1}} = c$

(E) $yx + \frac{x}{\sqrt{x^2 + 1}} = c$

17. The length of the curve $x(t) = e^t \cos t$, $y(t) = -e^t \sin t$ for $0 \leq t \leq 1$ is

(A) $2(e - 1)$

(B) $\sqrt{2}(e - 1)$

(C) e

(D) $2e$

(E) $\sqrt{2}$

18. The normal line to the graph of $3x^2 + 4x^2y + xy^2 = 8$ at $(1, 1)$ intersects the x-axis at $x =$

(A) $\frac{3}{2}$

(B) $-\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $-\frac{2}{3}$

(E) $-\frac{2}{5}$

19. Let $F(uv) = uv$ where $u = u(t)$ and $v = v(t)$. If $u(1) = 1$, $v(1) = 2$, $v'(1) = 1$ and $u'(1) = 2$, then $\frac{dF}{dt}$ at $t = 1$ is equal to

(A) 0

(B) 1

(C) 3

(D) 4

(E) 5

20. The number of points of discontinuity of the function

$$f(x)=\begin{cases} x+2 & \text{if } x\le 0 \\ 1 & \text{if } 0<x\le 2 \\ x-6 & \text{if } 2<x\le 5 \\ (6-x)^2 & \text{if } 5<x \end{cases}$$ is equal to

(A) 5

(B) 4

(C) 3

(D) 2

(E) 1

21. The solution set for the inequality $\frac{1}{x-2}<\frac{1}{x+3}$ is

(A) $(-3,-2)$

(B) $(-3,2)$

(C) $(2,3)$

(D) $(-2,2)$

(E) $(0,2)$

22. If $g\left(\frac{3+2x}{4}\right)=1-x$, for $-\infty<x<\infty$, then $g\left(\frac{7z-8}{4}\right)$ is equal to

(A) $-\frac{13+7z}{4}$

(B) $\frac{13}{2}-\frac{7z}{2}$

(C) $\frac{7-13z}{2}$

(D) $\frac{7+13z}{2}$

(E) $7z+13$

23. If $f'(e^x)=1+x$, then $f(x)=$

(A) $1+e^x+c$

(B) $1+\ln x+c$

(C) $\ln x+c$

(D) $x+\ln x+c$

(E) $x\ln x+c$

24. The iterated integral $\int_0^1\int_{\frac{y}{2}}^1 e^{x^2}\,dx\,dy$ can be expressed as

(A) $\int_0^1\int_0^{2x} e^{x^2}\,dy\,dx$

(B) $\int_{\frac{y}{2}}^1\int_0^1 e^{x^2}\,dy\,dx$

(C) $\int_0^1\int_0^{2y} e^{x^2}\,dx\,dy$

(D) $\int_0^1\int_0^{2y} e^{x^2}\,dy\,dx$

(E) $\int_0^1\int_y^1 e^{x^2}\,dy\,dx+\int_0^1\int_0^y e^{x^2}\,dx\,dy$

25. The series $\sum_{n=1}^{\infty}\frac{3^n}{n}(x-2)^n$ converges for x in the interval

(A) $\left(\frac{5}{3},\frac{7}{3}\right)$

(B) $\left(\frac{5}{3},\frac{7}{3}\right]$

(C) $\left[\frac{5}{3},\frac{7}{3}\right]$

(D) $\left[\frac{5}{3},\frac{7}{3}\right)$

(E) $\left(0,\frac{7}{3}\right)$

26. In R^3, an equation of the tangent plane to the surface $xz - yz^3 - yz^2 = 378$ at $(-3, 2, -6)$ is

(A) $2x + 60y + 65z = 516$

(B) $-2x + 60y + 65z + 516 = 0$

(C) $2x - 60y + 65z = 0$

(D) $x - 60y + z = 516$

(E) $2x - 60y + 65z + 516 = 0$

27. If $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, then $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$ is equal to

(A) $\frac{\pi^2}{12}$

(B) $\frac{\pi^2}{36}$

(C) $\frac{2\pi^2}{9}$

(D) $\frac{\pi^2}{7}$

(E) $\frac{\pi^2}{8}$

28. The area of the triangle in R^3 with vertices at $A = (2, 1, 5)$, $B = (4, 0, 2)$ and $C = (-1, 0, -1)$ is

(A) $\frac{475}{2}$

(B) 475

(C) $\sqrt{475}$

(D) $\frac{\sqrt{475}}{2}$

(E) $\sqrt{\frac{475}{2}}$

29. The order of the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 2 & 5 & 3 & 1 \end{pmatrix}$ is

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

30. Let T be a linear transformation from R^3 to R^3. If u and v are two orthogonal vectors in R^3, which of the following pairs of vectors must be orthogonal to one another?

(A) ru and sv for all real r and s

(B) $u + v$ and $u - v$

(C) Tu and u

(D) Tv and v

(E) Tu and Tv

31. If $x \in A \cap B$, which of the following is (are) true?

I. $x \in A$

II. $x \in B$

III. $x \in A' \cup B'$, where A' is the complement of A and B' is the complement of B

(A) I only

(B) II only

(C) I, II and III

(D) I and II only

(E) III only

32. Let R be the set of real numbers. Define $a * b = a + b + ab$ for a, b in R. The solution of $5 * x * 3 = 7$ is

(A) $\frac{7}{15}$

(D) $\frac{3}{2}$

(B) $\frac{15}{7}$

(E) $-\frac{2}{3}$

(C) $\frac{2}{3}$

33. If $(1 + 2x + 3x^2 + \ldots + (n+1)x^n \ldots)^2 = \sum_{k=0}^{\infty} b_k x^k$ for $|x| < 1$ then b_k is

(A) $k(k+1)(k+2)(k+3)$

(B) $\frac{k(k+1)(k+2)}{6}$

(C) $\frac{(k+1)(k+2)(k+3)}{6}$

(D) $\frac{(k+1)(k+2)}{3}$

(E) $(k+2)(k+3)$

34. Suppose $g'(x)$ exists for all real x and $g(a) = g(b) = g(c) = 0$ where $a < b < c$. The minimum possible number of zeros for $g'(x)$ is

(A) 1

(D) 4

(B) 2

(E) 5

(C) 3

35. $\int_0^{\frac{\pi}{2}} |\sin x - \cos x| \, dx =$

(A) $2\sqrt{2} - 2$

(D) $2\sqrt{2}$

(B) 0

(E) $2 - 2\sqrt{2}$

(C) 1

36. Which of the following are groups?

I. All integers under subtraction
II. All non-zero real numbers under division
III. All even integers under addition
IV. All integers which are multiples of 13 under addition

(A) I and II only

(D) IV only

(B) II and III only

(E) III and IV only

(C) III only

37. $(1+\sqrt{-1})^{8n} - (1-\sqrt{-1})^{8n}$ is equal to

(A) 2^{4n}

(B) $(-1)^{n+1}\,2^{4n}$

(C) 2^{4n+1}

(D) 0

(E) $-e^{-2}$

38. Given two vectors $\vec{U} = 2\vec{i} - 3\vec{j} + 5\vec{k}$, $\vec{V} = -\vec{i} + 4\vec{j} + 2\vec{k}$ then their vector product $\vec{U} \times \vec{V} =$

(A) $\vec{i} + \vec{j} + 7\vec{k}$

(B) $-2\vec{i} - 12\vec{j} + 10\vec{k}$

(C) $-26\vec{i} - 9\vec{j} + 5\vec{k}$

(D) $3\vec{i} + 4\vec{j} + 5\vec{k}$

(E) $3\vec{i} - 4\vec{j} - 5\vec{k}$

39. For what value(s) of p does the system of equations

$$\begin{aligned} px + y \quad &= 1 \\ x + py \quad &= 2 \\ y + pz &= 3 \end{aligned}$$

have no solution?

(A) 0, 1

(B) 1, −1

(C) −1

(D) −2

(E) 0, 1, −1

40. If the determinant of the matrix:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ x & 2 & 0 & 0 & 0 \\ x^2 & x^3 & 3 & 0 & 0 \\ x^3 & x^4 & x^5 & 4 & 0 \\ x^4 & x^5 & x^6 & x^7 & 0 \end{bmatrix}$$

is zero, how many values of x are possible?

(A) 0

(B) 1

(C) 2

(D) 3

(E) ∞

41. The nth derivative of $f(x) = \dfrac{1}{1-x^2}$ is

(A) $\dfrac{1}{(1-x^2)^{n+1}}$

(B) $\dfrac{x^n}{(1-x^2)^{n+1}}$

(C) $\dfrac{(-1)^n n!}{(1-x^2)^{n+1}}$

(D) $\dfrac{n!}{(1-x^2)^{n+1}}$

(E) $\dfrac{n!}{2}\left[\dfrac{1}{(1-x)^{n+1}} + \dfrac{(-1)^n}{(1+x)^{n+1}}\right]$

42. How many different partial derivatives of order k are possible for a function $f(x_1, \ldots, x_n)$ of n variables?

(A) 2^{n+k}

(B) $\begin{pmatrix} n+k-1 \\ n \end{pmatrix}$

(C) $\binom{n+k-1}{k}$ (D) $\binom{n+k}{k}$

(E) $\binom{n+k}{n-1}$

43. Let $T: C^{\infty}(R) \to C^{\infty}(R)$ be a linear map such that $T(e^{2x}) = \sin x$, $T(e^{3x}) = \cos 4x$ and $T(1) = e^{5x}$, where $C^{\infty}(R)$ is the vector space infinitely differentiable functions on the real numbers R. Then $T(4e^{2x} + 7e^{3x} - 5)$ is equal to

(A) $4 \sin x + 7 \cos 4x - 5e^{5x}$ (D) $7 \cos 4x - 5e^{5x}$

(B) $4 \sin x + 7 \cos 4x - 5$ (E) $\sin x + 7 \cos x - 5e^{5x}$

(C) $4 \sin x + 7 \cos 4x$

44. How many ways can 8 teachers be divided among 4 schools if each school must receive 2 teachers?

(A) 520 (D) 225

(B) 250 (E) 2^4

(C) 2520

45. Let C be the circle $|z| = 3$, described in counterclockwise orientation, and write

$$g(w) = \int_C^1 \frac{2z^2 - 2 - z}{z - w}\, dz$$

Then $g(2)$ is equal to

(A) 1 (D) $4\pi i$

(B) $2\pi i$ (E) $8\pi i$

(C) 0

46. Given that

$$\ln 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots$$

$$\text{then } -2 + 1 - \frac{2}{3} + \frac{2}{4} - \frac{2}{5} + \frac{2}{6} - \ldots$$

is equal to

(A) $-2 \ln 2$ (D) $\ln 3$

(B) $-\ln 2$ (E) $-3 \ln 3$

(C) $2 \ln 2$

47. Let

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ -3 & -2 & 1 & 0 \\ -4 & -3 & -2 & 2 \end{bmatrix}$$

be a 3 x 3 matrix viewed as a linear transformation from R^4 to R^4. What is the dimension of the eigenspace corresponding to the eigenvalue $\lambda = 1$?

(A) 4 (D) 1

(B) 3 (E) 0

(C) 2

48. Let T be a linear transformation from a vector space V of dimension 11 onto a vector space W of dimension 7. What is the dimension of the null space of T?

(A) 0 (D) 4

(B) 2 (E) 5

(C) 3

49. In an $x-y$ plane, if $\vec{a} = (x, y)$, $\vec{b} = (x', y')$, then their scalor product $\vec{a} \cdot \vec{b} =$

(A) $xx' + yy'$ (B) $xx' - yy'$

(C) $xy - x'y'$ (D) $xy' + x'y$

(E) $xy' - x'y$

50. What is the mean of the random variable $\underline{\overline{X}}$ whose distribution function is defined by

$$F(x) = \begin{cases} \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^x e^{-t^2}\, dt, & x > 0 \\ 0, & x \leq 0 \end{cases}$$

(A) $\frac{2}{\pi}$ (D) 0

(B) $\frac{1}{\sqrt{\pi}}$ (E) Does not exist

(C) $\sqrt{\frac{2}{\pi}}$

51. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$, a 2 x 2 matrix, which of the following is true?

(A) $A^2 - 3A = 0$ (D) $A^2 - 3A + I = 0$

(B) $I - 3A = 0$ (E) None of the above

(C) $A^2 + I = 0$

52. The equation of the tangent line to the graph of the equation $\begin{cases} x = t^3 - 4 \\ y = 2t^2 + 1 \end{cases}$ at $t = 2$ is

(A) $2x - 3y - 19 = 0$

(B) $2x - 3y + 19 = 0$

(C) $3x - 2y - 6 = 0$

(D) $3x - 2y + 6 = 0$

(E) $3x + 2y - 6 = 0$

53. If $f(0) = 1$, $f(2) = 3$ and $f'(2) = 5$, then $\int_0^1 x f''(2x)\, dx =$

(A) 0

(B) 1

(C) 2

(D) -1

(E) -2

54. Let P denote the product of any four consecutive integers. Then $1 + P$ is

(A) a multiple of 5

(B) a multiple of 4

(C) a prime number

(D) a complete square

(E) none of the above

55. The solution of the differential equation $y'' + 5y' + 6y = 0$ satisfying the initial conditions $y(o) = 0$ and $y'(0) = 1$ is

(A) $e^{2x} + e^{3x}$

(B) $e^{2x} - e^{3x}$

(C) $e^{-2x} + e^{3x}$

(D) $e^{-2x} + e^{-3x}$

(E) $e^{-2x} - e^{-3x}$

56. Which of the following functions is (are) analytic?

I. $\bar{z}$

II. $\bar{z} \sin z$

III. $z + \sin z$

IV. $z + \bar{z}$

V. $z e^z$

(A) I only

(B) II and I only

(C) III and V only

(D) IV only

(E) None of the above

57. For any positive integer n, $n^7 - n$ is divisible by...

(A) 4

(B) 6

(C) 7

(D) 14

(E) 18

Consider the permutation $f = (1478)(265)(39)$ in S_9. $f^{-1} =$

(A) (1874) (256) (39)

(B) (1874) (265) (39)

(C) (1847) (256) (39)

(D) (1874) (265) (39)

(E) None of the above

For what value of c is the function

$$f(x) = \begin{cases} cxe^{-x^2}, & 0 < x < \infty \\ 0 & , \text{elsewhere} \end{cases}$$

the probability density function of a random variable $\overline{X}$?

(A) 1

(B) 2

(C) 3

(D) π

(E) 2π

The negation of $\forall x \exists y \; (P(x,y) \wedge \neg Q(x,y))$ is

A) $\forall x \exists y \; (P(x,y) \rightarrow Q(x,y))$

B) $\forall x \exists y \; (Q(x,y) \rightarrow P(x,y))$

C) $\exists x \forall y \; (P(x,y) \rightarrow Q(x,y))$

D) $\exists x \forall y \; (Q(x,y) \rightarrow P(x,y))$

E) None of the above

61. If A is a countable subset of the interval [0, 1], then the Lebesque measure of A is equal to

(A) $\frac{1}{2}$

(B) 0

(C) $\frac{2}{3}$

(D) 3^{-1}

(E) None of the above

62. What is the interval of convergence of the series

$$\sum_{n=0}^{\infty} (3-x)(6x-7)^n \; ?$$

(A) $[1, \frac{4}{3}]$

(B) $(1, \frac{4}{3}]$

(C) $[1, \frac{4}{3})$

(D) $(1, \frac{4}{3})$

(E) $(0, \frac{4}{3})$

63. Given

$$A = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$$

with $\det(A) \neq 0$, the system of equations

$$x_1 + \frac{1}{2}x_2 + \frac{1}{3}x_3 + \frac{1}{4}x_4 = 1$$
$$\frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{4}x_3 + \frac{1}{5}x_4 = 2$$
$$\frac{1}{3}x_1 + \frac{1}{4}x_2 + \frac{1}{5}x_3 + \frac{1}{6}x_4 = 3$$
$$\frac{1}{4}x_1 + \frac{1}{5}x_2 + \frac{1}{6}x_3 + \frac{1}{7}x_4 = 4$$

has

(A) No solutions

(B) A unique solution

(C) 2 solutions

(D) 3 solutions

(E) Infinitely many solutions

64. Suppose $f(x) = x^4 + x^6 + \sin(x^2) + e^{x^3}$ and $f(x) = f_1(x) + f_2(x)$, where $f_1(x) = f_1(-x)$ and $f_2(-x) = -f_2(x)$. Then $f_2(x)$ is equal to

(A) $\frac{1}{2}\left(e^{x^3} - e^{-x^3}\right)$

(B) $\frac{1}{2}\left(e^{x^3} + e^{-x^3}\right)$

(C) $\frac{1}{2}(x^4 + x^6)$

(D) $\sin(x^2)$

(E) $x^4 + x^6 + \sin(x^2) + e^{x^3}$

65. If w is an n^{th} root of unity other than one, then the sum $w + w^2 + \ldots + w^{n-1}$ is equal to

(A) 1

(B) 0

(C) -1

(D) 3

(E) -2

66. If $P(A) = 0.7$, $P(B) = 0.5$ and $P([A \cup B]') = 0.1$ then $P(A \mid B)$ is

(A) $\frac{3}{7}$

(B) $\frac{3}{5}$

(C) $\frac{5}{7}$

(D) $\frac{7}{9}$

(E) 1

GRE MATHEMATICS TEST V

ANSWER KEY

23.	E	45.	E
24.	A	46.	A
25.	D	47.	D
26.	E	48.	D
27.	E	49.	A
28.	D	50.	B
29.	D	51.	D
30.	A	52.	B
31.	D	53.	C
32.	E	54.	D
33.	C	55.	E
34.	B	56.	C
35.	A	57.	C
36.	E	58.	A
37.	C	59.	B
38.	D	60.	C
39.	E	61.	B
40.	E	62.	D
41.	E	63.	B
42.	C	64.	A
43.	A	65.	C
44.	C	66.	B

GRE MATHEMATICS TEST V

DETAILED EXPLANATIONS OF ANSWERS

1. (E)

Let $L = \lim_{n \to \infty} \left(\prod_{m=2}^{n} \left(1 - \frac{1}{m}\right) \right) = \lim_{n \to \infty} \left(\prod_{m=2}^{n} \left(\frac{m-1}{m}\right) \right)$

Taking the natural logarithm of both sides we have

$$\ln L = \ln \lim_{n \to \infty} \left(\prod_{m=2}^{n} \frac{m-1}{m} \right) = \lim_{n \to \infty} \ln \left(\prod_{m=2}^{n} \left(\frac{m-1}{m}\right) \right)$$

We can interchange limit with ln because $\ln x$ is continuous. Using $\ln(ab) = \ln a + \ln b$ we get

$$\begin{aligned} \ln L &= \lim_{n \to \infty} \sum_{m=2}^{n} [\ln(m-1) - \ln m] \\ &= \lim_{n \to \infty} (\ln 1 - \ln 2) + (\ln 2 - \ln 3) + \ldots + (\ln(n-1) - \ln n) \\ &= \lim_{n \to \infty} (\ln 1 - \ln n) \\ &= \lim_{n \to \infty} (-\ln n) \\ &= -\infty, \end{aligned}$$

since $\ln 1 = 0$:

$$\Rightarrow L = \lim_{b \to \infty} e^{-b} = \lim_{b \to \infty} \frac{1}{e^{b}} = 0.$$