$$[f'(x)]^2 - [f(x)]^2 = [2\cosh x]^2 - [2\sinh x]^2$$
$$= 4.$$

3. (E)

The numerator is defined for all real numbers since it contains a cube root. The denominator is defined and nonzero except for $x = 6$. So, the domain is $R\backslash\{6\}$.

4. (E)

The determinant of M, denoted $|M|$, is given by

$$|M| = \begin{vmatrix} 1 & 2 \\ 3 & 9 \end{vmatrix} = 3.$$

For an $n \times n$ nonsingular matrix M, we have $M^{-1} = |M|^{-1}\,\text{adj}(M)$ and $|M^{-1}| = |M|^{-1}$ so that

$$|M^{-1}| = |\,|M|^{-1}\ \text{adj}(M)\,|$$
$$|M|^{-1} = (\,|M|^{-1})^n\ |\text{adj}(M)|$$
$$|\text{adj}(M)| = |M|^{n-1}$$

Since $|M| = 3$, $|\text{adj}(M)| = |M|^{2-1} = 3$.

5. (B)

Let g be any generator of a cyclic group G of order 8. The generators of G are of the form g^r where r is relatively prime to 8 (that is, the greatest common divisor of r and 8 is 1). The positive integers less than 8 and relatively prime to 8 are 1, 3, 5, 7. Therefore the four generators of G are g^1, g^3, g^5, and g^7.

6. (C)

An integrating factor for an ordinary differential equation of the form $M\,dx + N\,dy = 0$ is a function of the form $J(x, y)$ such that $JM\,dx + JN\,dy = 0$ is exact, that is,

$$\frac{\partial(JM)}{\partial y} = \frac{\partial(JN)}{\partial x}.$$

The given equation is not exact since

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = \frac{-2}{x} - 2xy\cos y$$
$$= \frac{-2}{x}\,(1 + x^2 y\cos y)$$
$$\neq 0$$

but since $r(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{-2}{x}$ is a function of x, an integrating factor is given by

$$J(x) = e^{\int r(x)\,dx}$$
$$= e^{-\int \frac{2}{x}\,dx}$$
$$= e^{-2\ln x}$$
$$= \frac{1}{x^2}.$$

7. (D)

We have $x = \sqrt{3 + x}$ or $x^2 = 3 + x$ so that $x^2 - x - 3 = 0$.

Using the quadratic formula, we obtain $x = \dfrac{1 \pm \sqrt{13}}{2}$.

We choose the positive square root, obtaining $x = \dfrac{1 + \sqrt{13}}{2}$ since x is positive.

8. (B)

In order for $f(x)$ to be a valid probability density function

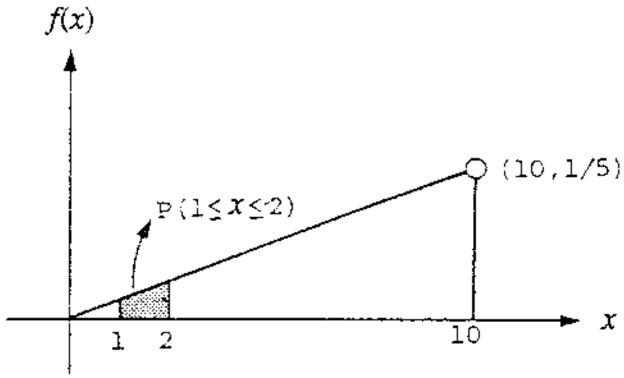
$$\int_{-\infty}^{+\infty} f(x)\, dx = 1.$$

Thus, $1 = \int_0^{10} cx\, dx = 50c$ so that $c = \frac{1}{50}$.

The probability that x is in [1, 2], denoted $P(1 \le x \le 2)$, is given by

$$P(1 \le x \le 2) = \int_1^2 \frac{x}{50}\, dx = \frac{1}{50}\left[\frac{x^2}{2}\right]_1^2 = \frac{3}{100}.$$

The graph of the probability density function is shown below.



9. (E)

The expected value of a function $g(x)$ of a random variable X, denoted $E[g(X)]$, is defined by

$$E[g(X)] = \int_{-\infty}^{+\infty} g(x)\, f(x)\, dx$$

where $f(x)$ is the probability function for X. We have

$$E[X^2] = \int_{-\infty}^{\infty} X^2 f(x) dx = \int_0^2 X^2\left(1 - \frac{x}{2}\right) dx$$

$$= \left[\frac{x^3}{3} - \frac{x^4}{8}\right]_0^2$$

$$= \frac{2}{3}.$$

10. (B)

An algebraic equation is of the form

$$a_N x^N + a_{N-1}x^{N-1} + \ldots + a_1 x + a_0 = 0$$

where N is a positive integer and $a_N, \ldots a_0$ are integers. ($a_N \ne 0$). The height h is defined by $h = N + |a_N| + \ldots + |a_0|$. For $h = 2$, we have $N = 1, |a_1| = 1$, and $|a_0| = 0$. This implies that $a_1 = \pm 1$ so that $x = 0$. Note that there are no other possibilities except for $N = 1$. We see that if $N = 0$ then the equation is of degree 0 and is of the form $a_0 = 0$, which is unacceptable, as we require $a_N \ne 0$. If $N = 2$, on the other hand, we obtain $h = 2 = N + 0 + 0 + \ldots + 0$, so $a_2 = 0 = a_1 = a_0$ which is again unacceptable.

11. (D)

A point (x, y) is in the unit ball if and only if $|x| + |y| < 1$. The boundary consists of the following lines:

a) $x \ge 0;\ y \ge 0:\quad x + y = 1$

b) $x \ge 0;\ y \le 0:\quad x - y = 1$

c) $x \le 0;\ y \ge 0:\ -x + y = 1$

d) $x \le 0;\ y \le 0:\ -x - y = 1$

These lines intersect at $(-1, 0)$, $(0, 1)$, $(1, 0)$, and $(0, -1)$.

12. (D)

Trying the given solutions:

$$-f(x^{-1}) = -\left(\frac{\frac{1}{x} + 1}{\frac{1}{x} - 1}\right) = \frac{x+1}{x-1} = f(x)$$

and $[f(-x)]^{-1} = \left[\frac{-x+1}{-x-1}\right]^{-1} = \frac{-x-1}{-x+1} = \frac{x+1}{x-1} = f(x)$.

But if we solve $y = f(x) = \frac{x+1}{x-1}$ for x:

$$y(x-1) = x+1$$

$$xy - x = y + 1$$

$$x = \frac{y+1}{y-1}$$

Thus $f^{-1}(x) = f(x)$. However,

$$f^{-1}(x^{-1}) = \frac{x^{-1}+1}{x^{-1}-1}$$

$$= \frac{1+x}{1-x}$$

$$= -f(x)$$

13. (E)

Let $\lambda_1, \lambda_2, \ldots \lambda_n$ be the eigenvalues of an $n \times n$ matrix M. The trace of M, denoted $tr(M)$, equals $\sum_{k=1}^{n} \lambda_k$. If p is a positive integer, then $tr(M^p)$ equals $\sum_{k=1}^{n} \lambda_k^p$. The eigenvalues of M are given by

$$\begin{vmatrix} 6-\lambda & 10 \\ -2 & -3-\lambda \end{vmatrix} = 0$$

which implies

$$-18 - 6\lambda + 3\lambda + \lambda^2 + 20 = 0$$

$$(\lambda - 1)(\lambda - 2) = 0$$

$$\lambda = 1,2$$

Thus $tr(M^5) = 1^5 + 2^5 = 33$.

14. (B)

The Cayley-Hamilton theorem guarantees that M will satisfy polynomial of degree 8. However, since M is idempotent $M^2 = M$ $M^2 - M = 0$. Thus M satisfies $p(x) = x^2 - x$. Assuming that M satisfi a linear polynomial (degree 1) equation $\hat{p}(x) = a_1 x + a_0 = 0$ impli that M is a scalar matrix, i.e.,

$$a_1 M + a_0 I_8 = 0$$

$$M = -\frac{a_0}{a_1} I_8 .$$

which contradicts the fact that M is nonscalar.

15. (C)

The incidence matrix for the graph G is a 5×5 matrix whos (i, j) entry equals the number of edges connecting x_i and x_j. Thus th matrix of G is given by

$$\begin{bmatrix} 0 & 2 & 0 & 0 & 1 \\ 2 & 0 & 3 & 1 & 0 \\ 0 & 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

16. (D)

A fixed point z must satisfy $z = w(z) = \frac{z-2}{z-1}$ which implie $z^2 - z = z - 2$. The solutions of $z^2 - 2z + 2 = 0$ are given by

$$z = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = 1 \pm i .$$

. (A)

The n^{th} roots of unity are solutions of the polynomial equation $-1 = 0$. For $n > 1$ and a leading coefficient of 1, the sum of the roots a polynomial is equal to the negative of the coefficient of the x^{n-1} rm. Hence the sum of the n^{th} roots of unity ($n > 1$) is zero. For another emonstration of this fact, let $1, \alpha, \ldots, \alpha^{n-1}$ denote the n^{th} roots of unity > 1). Then

$$1 + \alpha + \ldots + \alpha^{n-1} = \frac{1-\alpha^n}{1-\alpha} = 0 \text{ since } \alpha^n = 1 .$$

8. (D)

A Boolean variable is a variable whose value can be either 0 or . A Boolean function is a function whose variables (both independent nd dependent) are Boolean variables. The function $f(x, y, z) = x + + z - xy - xz$ is not a Boolean function since $f(1, 0, 1) = 2$.

9. (A)

The perimeter $P(x, y)$ for the rectangle shown in the figure below s $P(x, y) = 4x + 4y$. We want to maximize this quantity subject to the onstraint

$$\varnothing(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 .$$

ntroducing a Lagrange multiplier λ, we must maximize $L(x, y)$ $= P(x, y) + \lambda \varnothing(x, y)$. Taking the first order partial derivatives of $L(x, y)$ and setting them equal to zero yields

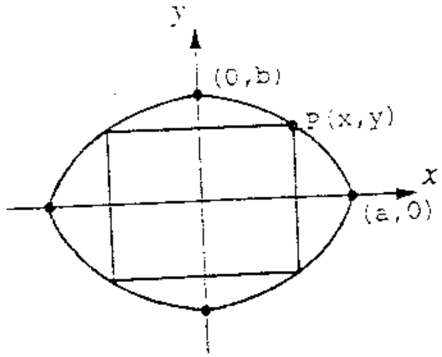
$$\frac{\partial L}{\partial x} = 4 + \lambda \frac{2x}{a^2} = 0$$

$$\frac{\partial L}{\partial y} = 4 + \lambda \frac{2y}{b^2} = 0 .$$

Thus $\frac{2x}{a^2} = \frac{2y}{b^2}$. Substituting $y = \frac{b^2 x}{a^2}$ into $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ yields $\frac{x^2}{a^2} + \frac{b^4 x^2}{a^4 b^2} = 1$, so that $x = \frac{a^2}{\sqrt{a^2 + b^2}}$ and

$$y = \frac{b^2}{\sqrt{a^2 + b^2}} .$$

The maximum perimeter is $4\sqrt{a^2 + b^2}$.



20. (D)

A one-to-one bicontinuous function h from a topological space (X, τ) onto a topological space (X', τ') is called a homeomorphism. A property P of sets of a topological space (X, τ) is called topological if it is invariant under homeomorphisms. To see that the property of being an accumulation point is topological, let x be an accumulation point of a set $S \subseteq X$ and consider $h(x) \in h(S) \subseteq X'$. Let $D_{x'}$ be an open set in X' containing $h(x)$. Then $D_{x'} = h^{-1}(D_{x'})$ is an open set in X containing x. Since x is an accumulation of point of S, there exists an $\bar{x} \in S \cap D_X$ such that $x \neq \bar{x}$. Thus $h(\bar{x}) = h(S) \cap D_{X'}$ distinct from $h(x)$, which implies that $h(x)$ is an accumulation point of $h(S)$.

21. (C)

According to Cauchy's integral formula we have

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a}\, dz ,$$

where C is a closed contour which includes a, and f is analytic within and on C. Now, if we let

$$f(z) = \frac{(\cos z)}{(z-\pi)},$$

and $a = 0$, we will have

$$f(a) = f(0) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z}\, dz$$

$$= \frac{1}{2\pi i} \oint_C \frac{\cos z}{z-\pi}\, dz$$

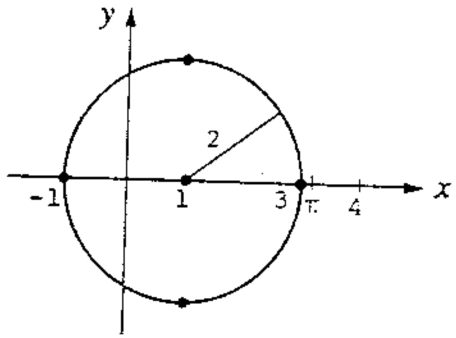
$$= \frac{\cos 0}{0-\pi}$$

$$= \frac{1}{-\pi}$$

$$\Rightarrow \oint_C \frac{\cos z}{z-\pi}\, dz = \frac{-1}{\pi}(2\pi i)$$

$$= -2i$$

Note that we can use the Cauchy integral formula for the circle $C: |z-1| = 2$ since it does not include the point $z = \pi$.



22. (B)

Propositional algebra is similar to set algebra with union ($\cup$) replaced with disjunction ($\vee$), intersection ($\cap$) replaced with conjunction ($\wedge$) complementation (^{c}) replaced with negation ($\sim$), the universal set (Ω) replaced with tautology (1), and the empty set ($\varnothing$) replaced with absurdity (0). Also

$$\sim(\forall x \in S) w(x) \Leftrightarrow (\exists x \in S)[\sim w(x)] \text{ and}$$

$$\sim(\exists x \in S) w(x) \Leftrightarrow (\forall x \in S)[\sim w(x)]$$

for an open statement $w(x)$ relative to S. We have

$$\sim(\exists x \in S)[(p(x) \vee q(x)) \wedge r(x)]$$

$$\Leftrightarrow (\forall x \in S)\{\sim[(p(x) \vee q(x)) \wedge r(x)]\}$$

$$\Leftrightarrow (\forall x \in S)\{[\sim(p(x) \vee q(x))] \vee [\sim r(x$$

$$\Leftrightarrow (\forall x \in S)\{[(\sim p(x)) \wedge (\sim q(x))] \vee [\sim r$$

23. (B)

We have

$$\lim_{n\to+\infty} \frac{x_{n+1}}{x_n} = \lim_{n\to+\infty} \frac{(n+1)^{n+1} n!}{(n+1)!\, n^n}$$

$$= \lim_{n\to+\infty} \frac{(n+1)^n (n+1)\, n!}{n^n\, (n+1)!}$$

$$= \lim_{n\to+\infty} \left(1 + \frac{1}{n}\right)^n$$

$$= e$$

24. (A)

By the chain rule, the derivative of a function defined b

$$\phi(x, a(x), b(x)) = \int_{a(x)}^{b(x)} g(x, t)\, dt$$

is given by

$$\frac{d\phi(x, a(x), b(x))}{dx} = \frac{\partial\phi}{\partial x} + \frac{\partial\phi}{\partial a}\frac{da(x)}{dx} + \frac{\partial\phi}{\partial b}\frac{db(}{dx}$$

$$\int_{a(x)}^{b(x)} \frac{\partial g(x,t)}{dx}\, dt + g(x,b(x))\,\frac{db(x)}{dx} - g(x,a(x))\,\frac{da(x)}{dx}.$$

...at in evaluating $\frac{\partial \varnothing}{\partial a}$ and $\frac{\partial \varnothing}{\partial b}$, a and b must be considered as ...ndent variables. Therefore, before taking the derivative with ... to either a or b, we must replace $a(x)$ and $b(x)$ by a and b in the ...on of $\varnothing(x, a(x), b(x))$. Thus

$$\frac{df}{dx} = \int_x^0 \left[\frac{-t \sin xt}{t}\right] dt + 0 - \frac{\cos x^2}{x}$$

$$= \left[\frac{\cos xt}{x}\right]_x^0 - \frac{\cos x^2}{x}$$

$$= \frac{1}{x} - \frac{\cos x^2}{x} - \frac{\cos x^2}{x}$$

$$= \frac{1}{x}[1 - 2\cos x^2]$$

$$= -\frac{\cos 2x^2}{x}.$$

...B)

... directional derivative of the function $P(x, y)$ along the ...ugh (x, y) and parallel to the unit vector $\vec{u}$ is equal to $\vec{P} \cdot \vec{u}$. Therefore, since the values $|\nabla \vec{P}|$ and $|\vec{u}| = 1$ ...ant, the directional derivative of $\frac{\partial f}{\partial u}$ is maximized when the ...tween $\nabla \vec{P}$ and $\vec{u}$ is zero. Therefore, the maximum value ...rectional derivative at $P(x, y)$ equals the magnitude of the

$$\nabla f(x,y) = \frac{\partial f}{\partial x}\, i + \frac{\partial f}{\partial y} j$$

. We have

$$\frac{\partial f}{\partial x} = xye^{xy} + e^{xy} - y\sin x \;;\; \frac{\partial f(0,1)}{\partial x} = 1$$

$$\frac{\partial f}{\partial y} = x^2 e^{xy} + \cos x \;;\; \frac{\partial f(0,1)}{\partial y} = 1$$

Thus $\nabla f(0,1) = \vec{i} + \vec{j}$ so that $|\nabla f(0,1)| = \sqrt{2}$.

26. (C)

Let n and k represent positive integers with k satisfying $1 \le k \le n$. An ordered partition of n into k parts is a decomposition of n into the sum of k positive integers:

$$n = x_1 + x_2 + x_3 + \ldots + x_k.$$

For a fixed k, the number of ordered partitions of n into k parts is the number of distinct ways of placing $\bar{k} = k - 1$ identical markers in the $\bar{n} = n - 1$ spaces between a row of n ones. The first marker can be placed in $\bar{n}$ ways, the second in $\bar{n} - 1$ ways, ..., and the last marker in $\bar{n} - [\bar{k} - 1]$ ways. The number of ways is therefore $\bar{n}(\bar{n} - 1) \ldots (\bar{n} - \bar{k} + 1)$.

Since the markers are identical, in each way of placing them we can switch the markers around without changing the chosen spaces and still have the same way of placing. Therefore, each group of $k!$ ways of placing the k markers, as long as the spaces chosen are still the same, are actually identical. Hence we must divide the above result by $k!$ to arrive at the correct answer. That is, the number of distinct ways is:

$$\frac{\bar{n}(\bar{n} - 1) \ldots (\bar{n} - [\bar{k} - 1])}{\bar{k}!} = \frac{\bar{n}!}{\bar{k}!(\bar{n} - \bar{k})!} = \binom{\bar{n}}{\bar{k}},$$

which is the binomial coefficient. Since we can have ordered partitions of 5 into 1, 2, 3, 4 or 5 parts (that is, we can decompose 5 as a sum of 1, 2, 3, 4 or 5 positive integers), the number of ordered partitions of 5 is

$$\sum_{k=0}^{4} \left[\frac{4}{k} \right] = (1+1)^4 = 2^4 = 16.$$

27. (D)

The Wronskian is given by

$$W(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ f_1'(x) & f_2'(x) \end{vmatrix}$$

$$= \begin{vmatrix} x^2 \sin x & x^2 \cos x \\ 2x \sin x + x^2 \cos x & 2x \cos x - x^2 \sin x \end{vmatrix}$$

$$= 2x^3 \sin x \cos x - x^4 \sin^2 x - 2x^3 \sin x \cos x - x^4 \cos^2 x$$

$$= -x^4 .$$

28. (A)

Assuming a solution of the form $y_k = r^k$, we obtain

$$r^{k+2} - r^{k+1} - 2r^k = 0$$

$$r^k(r-2)(r+1) = 0$$

so that $r = -1, 2$. Hence

$$y_k = c_1(-1)^k + c_2 2^k$$

and since $y_0 = 9$ and $y_1 = -12$, we have

$$c_1 + c_2 = 9$$

$$-c_1 + 2c_2 = -12$$

This implies $c_1 = 10$ and $c_2 = -1$ which implies $y_k = 10(-1)^k - 2^k$. For $k = 6$, we obtain $y_6 = -54$. Note that in the general form of a second order homogeneous recursion equation, we have $y_{k+2} + Ay_{k+1} + By_k = 0$. The general form of the solution of this equation is $c_1 r_1^{\,k} + c_2 r_2^{\,k} = y_k$. To evaulate r_1 and r_2, we can insert the special solution r^k directly into the equation and get:

$$r^{k+2} + Ar^{k+1} + Br^k = 0$$

$$\Rightarrow r^k(r^2 + Ar + B) = 0$$

$$\Rightarrow r^2 + Ar + B = 0.$$

We observe that if this quadratic equation has 2 distinct roots, r_1 a[nd] r_2, then any expression of the form $c_1 r_1^{\,k} + c_2 r_2^{\,k}$ will be a solution [of] the equation, where c_1, c_2 are arbitrary constants, because of t[he] linearity of the equation.

29. (C)

Let N represent a positive integer and write N as

$$N = u_0 + 10u_1 + 10^2 u_2 + 10^3 u_3 + \dots + 10^n u_n .$$

Recalling that $a \equiv b \pmod{c}$ means that $a - b$ is divisible by c, w[e] have that $u_j 10^j \equiv u_j \pmod 9$ for $0 \le j \le n$. Thus $N \equiv u_0 + u_1 + \dots + u_n \pmod 9$ which implies $N -$ (sum of digits of N) is divisible by 9. Therefore N is divisible by 9 if and only if the sum of its digits i[s] divisible by 9. Since $(3+2+2+4+4+6+6)/9 = 3$, 3224466 i[s] divisible by 9.

30. (C)

The possible inflection points of f occur where $f''(x) = 0$ or where $f''(x)$ does not exist. We have

$$f'(x) = \frac{x\left(\frac{1}{x}\right) - \ln x}{x^2} = \frac{1 - \ln x}{x^2}$$

$$f''(x) = \frac{x^2\left(\frac{-1}{x}\right) - 2x(1 - \ln x)}{x^4} = \frac{2\ln x - 3}{x^3}$$

etting $f''(x) = 0$ implies that $x = e^{3/2}$. There are no elements in he domain of $f(x)$ such that $f''(x)$ does not exist. The function f is oncave downward on $(0, e^{3/2}]$ and concave upward on $[e^{3/2}, +\infty)$. Thus, there is an inflection point at $x = e^{3/2}$.

1. (B)

We first reduce M to echelon form using elementary row operations:

$$M \xrightarrow[\substack{-R_1+R_2\to R_2 \\ R_1+R_2\to R_3 \\ 2R_1+R_4\to R_4}]{} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -4 & 3 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 4 & -1 & 3 \end{bmatrix}$$

$$\xrightarrow[R_3 \leftrightarrow R_4]{} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -4 & 3 & 1 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow[R_2+R_3\to R_3]{} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -4 & 3 & 1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\xrightarrow[\substack{-R_2\to R_2 \\ -\frac{1}{2}R_3+R_4\to R_4}]{} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 4 & -3 & -1 \\ 0 & 0 & 2 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow[\frac{1}{2}R_3\to R_3]{} \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 4 & -3 & -1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} = M_e$$

Since M_e has three linearly independent row vectors, the rank of M is three. This implies that the null space has dimension $4 - 3 = 1$.

32. (C)

For a ring R, the radical is the set of nilpotent elements of R, that is, the set $\{r \in R \mid r^n = 0 \text{ for some } n \in Z^+\}$. The powers of the elements of $Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ are

0: 0, 0, 0, 0, ...
1: 1, 1, 1, 1, ...
2: 2, 4, 0, 0, ...
3: 3, 1, 3, 1, ...
4: 4, 0, 0, 0, ...
5: 5, 1, 5, 1, ...
6: 6, 4, 0, 0, ...
7: 7, 1, 7, 1, ...

Thus the radical of Z_8 is $\{0, 2, 4, 6\}$.

33. (D)

Using the identities $\sin x = \dfrac{e^{ix} - e^{-ix}}{2i}$ and $\cos x = \dfrac{e^{ix} + e^{-ix}}{2}$, we obtain

$$\sin^3 x \cos^2 x = \left[\frac{e^{ix} - e^{-ix}}{2i}\right]^3 \left[\frac{e^{ix} + e^{-ix}}{2}\right]^2$$

$$= \frac{1}{-32i}\left[e^{2xi} - e^{-2xi}\right]^2 \left[e^{xi} - e^{-xi}\right]$$

$$= \frac{1}{-32i}\left[e^{4xi} - 2 + e^{-4xi}\right]\left[e^{xi} - e^{-xi}\right]$$

$$= \frac{1}{-32i}\left[e^{5xi} - 2e^{xi} + e^{-3xi} - e^{3xi} + 2e^{-xi} - e^{-5xi}\right]$$

$$= \frac{1}{16}\left[2\sin x + \sin 3x - \sin 5x\right]$$

34. (C)

Since f is continuous on $[0, 2\pi]$, an absolute maximum exists. It must occur at the endpoints $[(0, -1); (2\pi, -1)]$ or at an interior point where $f'(x) = 0$. Setting $f'(x) = -2\sin 2x + 2\sin x$ equal to zero implies $\sin x\,(1 - 2\cos x) = 0$, so that $x = \pi/3, \pi, 5\pi/3$. Since $f(\pi/3) = -3/2$, $f(\pi) = 3$, $f(5\pi/3) = -3/2$, $f(0) = -1$ and $f(2\pi) = -1$, the absolute maximum of f occurs at $x = \pi$.

35. (C)

A number λ is called an eigenvalue for a matrix M if there exists a nonzero vector X such that $MX = \lambda X$. This implies that the determinant $|\lambda I - M| = 0$. Thus

$$\begin{vmatrix} \lambda - 1 & 3 \\ 2 & \lambda - 2 \end{vmatrix} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0\ ;\ \lambda = -1, 4$$

For $\lambda = -1$, $[\lambda I - M]X = 0$ yields

$$\begin{bmatrix} -2x_1 & + & 3x_2 \\ 2x_1 & - & 3x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

so that $2x_1 - 3x_2 = 0$. Therefore $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is an eigenvector for $\lambda = -1$. The solution also follows from the fact that $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is an eigenvector:

$$\lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = -\begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

which implies $\lambda = -1$.

36. (D)

We can rearrange each term in the sum as follows:

$$\frac{1}{n^2 + n + 1} = \frac{(n+1) - n}{1 + n(n+1)}.$$

Now if we define $\tan a_n = n$, then we will have

$$\frac{1}{n^2 + n + 1} = \frac{\tan a_{n+1} - \tan a_n}{1 + \tan a_n \tan a_{n+1}} = \tan(a_{n+1} - a_n).$$

So $\arctan\left(\frac{1}{n^2 + n + 1}\right) = \arctan(\tan(a_{n+1} - a_n)) = a_{n+1} - a_n$

$$\text{So } \sum_{n=1}^{m} \arctan\left(\frac{1}{n^2 + n + 1}\right) = \sum_{n=1}^{m} (a_{n+1} - a_n)$$

$$= (a_2 - a_1) + (a_3 - a_2) + \ldots + (a_{m+1} - a_m)$$

$$= a_{m+1} - a_1$$

$$= \arctan(m + 1) - \arctan 1$$

$$= \arctan(m + 1) - \frac{\pi}{4}$$

37. (C)

The conjugates of $\sqrt{\sqrt{3} + 1}$ consist of the set of all zeros o the irreducible polynomial of $\sqrt{\sqrt{3} + 1}$ over the rational numbers We first determine a polynomial over the rationals for which $\sqrt{\sqrt{3} + 1}$ is a zero:

$$x = \sqrt{\sqrt{3} + 1}$$

$$x^2 = \sqrt{3} + 1$$

$$p(x) = x^4 - 2x^2 - 2$$

We now use the Eisenstein test to establish the irreducibility of $p(x)$ = $x^4 + 2x^2 - 2$ over the rational numbers. Firstly, $p(x)$ is an element o the set of polynomials with integer coefficients. Secondly, $a_2 = 1 \equiv$ (mod 2), $a_1 = -2 \equiv 0 \pmod 2$, $a_0 = -2 \equiv 0 \pmod{2^2}$. Therefore $p(x$ is irreducible over the rational numbers. The conjugates of $\sqrt{\sqrt{3} + 1}$ are the zeros of $p(x)$:

$$x^4 - 2x^2 - 2 = 0$$

$$x^2 = \frac{2 \pm \sqrt{4+8}}{2} = 1 \pm \sqrt{3}$$

$$x = \pm\sqrt{1+\sqrt{3}}, \pm\sqrt{1-\sqrt{3}}.$$

38. (E)

We have

$2^1 > 1^2$ is true
$2^2 > 2^2$ is false
$2^3 > 3^2$ is false
$2^4 > 4^2$ is false
$2^5 > 5^2$ is true

The inequality is true for $n = 5$. Assume $2^n > n^2$ for $n = k$. We will will show that $2^{k+1} > (k+1)^2$. Consider $f(x) = 2^x - 2x - 1$ so that $f'(x) = 2^x \ln 2 - 2$. We have $f(5) > 0$ and $f'(x) > 0$ for $x \in [5, +\infty)$ which implies $2^k > 2k + 1$ for $k \geq 5$. Since $2^k > k^2$, we have

$$2^k + 2^k > k^2 + 2k + 1$$

$$2^{k+1} > (k+1)^2.$$

39. (A)

For a nonstrictly determined two player (P_1, P_2) game G with payoff matrix M

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \end{array}$$

the minimax value v is given by $v = \frac{|M|}{d}$ where $d = (a_{11} + a_{22}) -$

Thus $v = \frac{5}{(1+3)-(-1+2)} = \frac{5}{3}$. This game favors player P_1 to the extent that it will, on the average, pay him 5/3 units/game.

40. (C)

The first Newton approximation can be obtained from the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

with $n = 0$. Since $f'(x) = 3x^2 - 2$, we have

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{4}{10} = \frac{8}{5}.$$

41. (B)

Note that $|x-2| = \begin{cases} -(x-2) & \text{if } x \leq 2 \\ (x-2) & \text{if } x > 2 \end{cases}$

Therefore

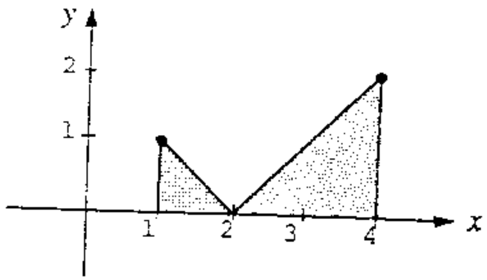
$$\int_1^4 |x-2|\, dx = -\int_1^2 (x-2)\, dx + \int_2^4 (x-2)\, dx$$

$$= -\left[\frac{x^2}{2} - 2x\right]_1^2 + \left[\frac{x^2}{2} - 2x\right]_2^4$$

$$= -\left[(2-4) - \left(\frac{1}{2} - 2\right)\right] + [(8-8) - (2-4)]$$

$$= \frac{5}{2}.$$

The graph of $|x - 2|$ on [1, 4] is shown following.



42. (B)

For a group of order n, a Sylow p-subgroup has order p^k where k is the largest positive integer such that p^k divides n. Since $72 = 2^3\,3^2$, a Sylow 3-subgroup has 9 elements.

43. (D)

A set G, together with a binary operation $*$ is called a group, denoted $(G, *)$, if

1) the binary operation $*$ is associative; $f, g, h \in G$ implies $(f * g) * h = f*(g * h)$,

2) G contains an identity element: there exists $e \in G$ such that $e * g = g * e = g$ for all $g \in G$.

3) Each element of G has an inverse: if $g \in G$, there exists $g' \in G$ such that $g * g' = g' * g = e$.

The set $G = R\backslash\{0\}$ together with the binary operation $a * b = |a|\,b$ does not form a group. The number 1 is a "right" identity element, but it is not a "left" identity element:

1) $1 * b = b$ for all $b \in G$

2) $a * 1 \neq a$ for $a \in G$ when $a < 0$

44. (C)

We have

$$M^2 = \begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}^2 = \begin{bmatrix} 8 & 16 \\ 4 & 8 \end{bmatrix} = 4M$$

so that $M^6 = (4M)^3 = 4^3 M^2 M = 4^4 M M = 4^5 M$.

45. (B)

The total number of selections possible is the number of ways of selecting 10 graduate students from 15 applicants which is $\begin{bmatrix} 15 \\ 10 \end{bmatrix}$. Since the selection process was random, the probability of any selection is $\dfrac{1}{\begin{bmatrix} 15 \\ 10 \end{bmatrix}}$. We must determine the number of selections which include 4 of the 5 "best students." Firstly, 4 of the possible 10 people selected must be selected from the 5 "best students." This can be accomplished in $\begin{bmatrix} 5 \\ 4 \end{bmatrix}$ ways. The other $10 - 4 = 6$ people must come from the other $15 - 5 = 10$ applicants. This can be accomplished in $\begin{bmatrix} 10 \\ 6 \end{bmatrix}$ ways. Thus, there are $\begin{bmatrix} 5 \\ 4 \end{bmatrix}\begin{bmatrix} 10 \\ 6 \end{bmatrix}$ ways of selecting 4 of the 5 "best students." Hence, the probability of selecting 4 of the 5 "best students" is

$$\frac{\begin{bmatrix} 5 \\ 4 \end{bmatrix}\begin{bmatrix} 10 \\ 6 \end{bmatrix}}{\begin{bmatrix} 15 \\ 10 \end{bmatrix}} = \frac{5!}{4!\,1!}\;\frac{10!}{6!\,4!}\;\frac{10!\,5!}{15!} = \frac{50}{143}$$

46. (A)

Since $r^2 = x^2 + y^2$, $\sin\theta = y/r$, and $\cos\theta = x/r$, we have

$$r = 2y/r - x/r$$
$$r^2 = 2y - x$$
$$x^2 + y^2 + x - 2y = 0$$

47. (B)

Set $x = 0.0259\overline{259}$. Then $9990x = 10000x - 10x = 259.\overline{259} - 0.\overline{259}$ so that

$$x = \frac{259}{9990} = \frac{7}{270}. \text{ Thus } 2.0259\overline{259} = \frac{547}{270}.$$

48. (D)

The Maclaurin series for e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$.

Therefore

$$xe^{-x^2} = x\left[1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} + \ldots + \frac{(-1)^n x^{2n}}{n!} + \ldots\right],$$

so that the general term is given by $\frac{(-1)^n x^{2n+1}}{n!}$.

49. (C)

The solution set for the inequality is equivalent to finding where the function

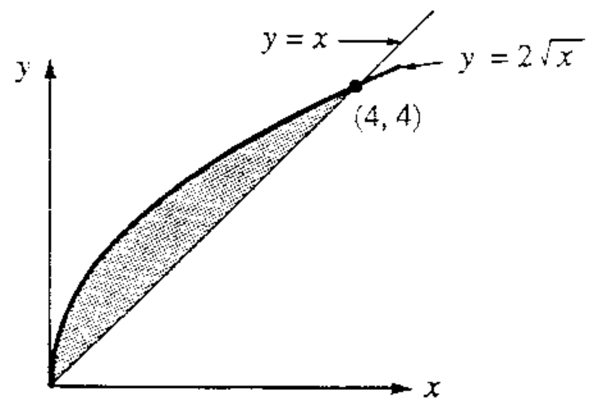
$$f(x) = x - \frac{3}{x} - 2 = \frac{(x-3)(x+1)}{x}$$

is positive. The figure below shows the x-axis subdivided into regions where f is continuous and never zero; we always omit endpoints. Thus f has the same sign throughout each subinterval; the signs are shown below. The solution of the inequality is $(-1,0) \cup (3,+\infty)$.

50. (D)

The x-coordinates of the points of intersection of the curves are solutions of $x = 2\sqrt{x}$ which implies $x = 0,4$. The region is shown below. The volume generated is given by

$$\text{Volume} = \pi\int_0^4 \left[(2\sqrt{x})^2 - (x)^2\right] dx$$
$$= \pi\left[2x^2 - \frac{x^3}{3}\right]_0^4$$
$$= \frac{32\pi}{3}.$$



51. (D)

Using the associative law for intersection, we can write:

$$[A \cap (A \cap B^C)] \cap B]^C = [A \cap A \cap B \cap B^C]^C$$
$$= [(A \cap A) \cap (B \cap B^C)]^C$$
$$= [A \cap (\varnothing)]^C$$
$$= \varnothing^C$$
$$= U .$$

52. (D)

The cross product of $\vec{u} \times \vec{v}$ is equal to the determinant

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix} = (\vec{i} + 4\vec{k} + 3\vec{j}) - (-\vec{k} + 6\vec{i} - 2\vec{j})$$
$$= -5\vec{i} + 5\vec{j} + 5\vec{k} .$$

53. (B)

The group $Z_2 \times Z_4$ contains eight elements

$$Z_2 \times Z_8 = \{(0,0), (0,1), (0,2), (0,3), (1,0), (1,1), (1,2), (1,3)\}$$

The cyclic subgroup <(1,1)> is given by

$$<(1,1)> = \{(1,1), (0,2), (1,3), (0,0)\}$$

Since

$$(0,1) + <(1,1)> = \{(1,2), (0,3), (1,0), (0,1)\}$$

there are two left cosets.

54. (B)

Let G be an abelian group with order n. Then G is isomorphic to the products of the form

$$Z_{(p_1{}^{n_1})} \times Z_{(p_2{}^{n_2})} \times \ldots \times Z_{(p_k{}^{n_k})} ,$$

Where the p_j's, not necessarily distinct, are the primes in the factorization of n and $(p_1{}^{n_1})(p_2{}^{n_2}) \ldots (p_k{}^{n_k}) = n$. Here Z_n denotes the cyclic group of $\{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition modulo n. For $n = 36 = 2^2\, 3^2$, we have G isomorphic to

$$Z_4 \times Z_9 = Z_{(2^2)} \times Z_{(3^2)}$$
$$Z_2 \times Z_2 \times Z_9 = Z_{(2^1)} \times Z_{(1^1)} \times Z_{(3^2)}$$
$$Z_4 \times Z_3 \times Z_3 = Z_{(2^2)} \times Z_{(3^1)} \times Z_{(3^1)}$$
$$Z_2 \times Z_2 \times Z_3 \times Z_3 = Z_{(2^1)} \times Z_{(2^1)} \times Z_{(3^1)} \times Z_{(3^1)} .$$

55. (C)

The sum of $a + ar + ar^2 + \ldots + ar^{n-1} + \ldots$ is $\dfrac{a(1-r^n)}{1-r}$.

We have

$$\sum_{j=0}^{10} (-i)^j = 1 - i + i^2 - i^3 + \ldots + i^{10}$$

so that $a = 1$, $r = -i$, and $n = 11$. Thus

$$\sum_{j=0}^{10} (-i)^j = \frac{1-(-i)^{11}}{1+i} = \frac{1+i}{1-i}\,\frac{1-i}{1-i} = -i .$$

56. (D)

The number 1 is the identity element R. An element $u = a + ib$ in R is a unit if there exists $v = c + id$ in R such that $u\,v = 1$. If u is a unit, then $\overline{u} = a - ib$ is also a unit since $\overline{uv} = 1$. We have

$$1 = uv = \overline{uv} \Rightarrow 1 = uv(\overline{uv}) = (u\overline{u})(v\overline{v})$$

$$= |u|^2|v|^2 = (a^2 + b^2)(c^2 + d^2)$$

Since a, b, c, d are integers and $a^2 + b^2 \neq 0$, we know that $a^2 + b^2 = 1$. The solutions are $a = 0$, $b = \pm 1$ and $a = \pm 1$, $b = 0$ which implies that the units are $\pm 1, \pm i$.

57. (C)

We have

$$3x + 11 \equiv 20 \pmod{12}$$
$$3x \equiv 9 \pmod{12}$$
$$x \equiv 3 \pmod{4}$$

The numbers in the set $\{\ldots, -9, -5, -1, 3, 7, 11, 15, \ldots\}$ satisfy $3x + 11 \equiv 20 \pmod{12}$. Each of these numbers is in one of the following equivalence classes:

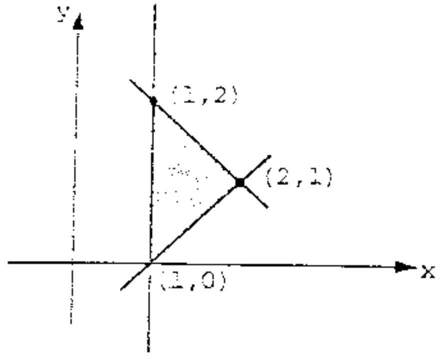
$$\langle 3 \rangle = \{\ldots, -9, 3, 15, 27, \ldots\}$$
$$\langle 7 \rangle = \{\ldots, -5, 7, 19, 31, \ldots\}$$
$$\langle 11 \rangle = \{\ldots, -1, 11, 23, 35, \ldots\}$$

58. (D)

The region R is shown below. Since R is convex and $f(x, y)$ is linear, the maximum of f occurs at a corner point. We have

$$f(1,0) = -2$$
$$f(2,1) = -1$$
$$f(1,2) = 4.$$

Therefore the maximum is 4.



59. (B)

We have

$$\begin{array}{r} x^2 - x + 1 \\ x^3 - 1 \overline{\smash{\big)} x^5 - x^4 + x^3 + 0x^2 + x} \\ x^5 \qquad\qquad - x^2 \\ \hline -x^4 + x^3 + x^2 + x \\ -x^4 \qquad\qquad + x \\ \hline x^3 + x^2 + x \\ x^3 \qquad\qquad - 1 \\ \hline x^2 + 1 \end{array}$$

so that $\dfrac{x^5 - x^4 + x^3 + x}{x^3 - 1} = x^2 - x + 1 + \dfrac{x^2 + 1}{x^3 - 1}$.

As $|x| \to +\infty$, $\dfrac{x^2 + 1}{x^3 - 1} \to 0$ so that $f(x) = x^2 - x + 1$.

60. (A)

Let F represent the number of faces, E the number of edges, and V the number of vertices of an ordinary polyhedron. Euler's theorem states that $F - E + V = 2$. Thus $12 - 17 + V = 2$ so that $V = 7$.

then $\bar{u} = a - ib$ is also a unit since $\overline{uv} = 1$. We have

$$1 = uv = \overline{uv} \Rightarrow 1 = uv(\overline{uv}) = (u\bar{u})(v\bar{v})$$

$$= |u|^2|v|^2 = (a^2 + b^2)(c^2 + d^2)$$

Since a, b, c, d are integers and $a^2 + b^2 \neq 0$, we know that $a^2 + b^2 = 1$. The solutions are $a = 0$, $b = \pm 1$ and $a = \pm 1$, $b = 0$ which implies that the units are $\pm 1, \pm i$.

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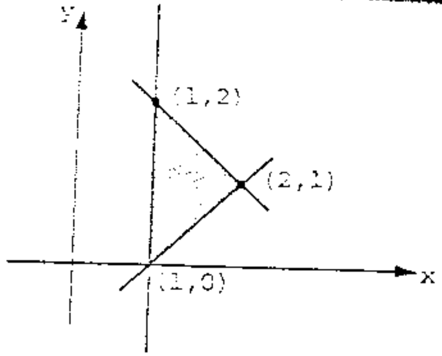


59. (B)

We have

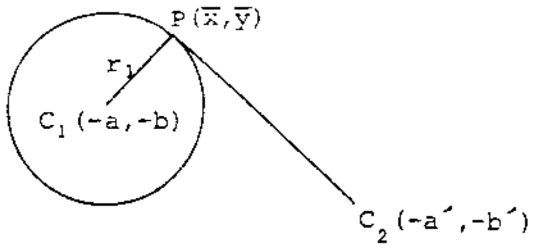
$$\begin{array}{r} x^2 - x + 1 \\ x^3 - 1 \overline{\big)\, x^5 - x^4 + x^3 + 0x^2 + x} \\ x^5 \qquad\qquad - x^2 \\ \hline -x^4 + x^3 + x^2 + x \\ -x^4 \qquad\qquad + x \\ \hline x^3 + x^2 + x \\ x^3 \qquad\quad - 1 \\ \hline x^2 + 1 \end{array}$$

so that $\dfrac{x^5 - x^4 + x^3 + x}{x^3 - 1} = x^2 - x + 1 + \dfrac{x^2 + 1}{x^3 - 1}$.

As $|x| \to +\infty$, $\dfrac{x^2 + 1}{x^3 - 1} \to 0$ so that $f(x) \approx x^2 - x + 1$.

60. (A)

Let F represent the number of faces, E the number of edges, and V the number of vertices of an ordinary polyhedron. Euler's theorem states that $F - E + V = 2$. Thus $12 - 17 + V = 2$ so that $V = 7$.



66. (A)

Using elementary row operations, we transform $[M \mid I]$ into $[I \mid M^{-1}]$:

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 0 & -1 & 2 & 0 & 1 & 0 \\ 4 & 3 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 4 & 3 & 1 & 0 & 0 & 1 \\ 0 & -1 & 2 & 0 & 1 & 0 \end{array}\right] \quad R_2 \Leftrightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -5 & -2 & 0 & 1 \\ 0 & -1 & 2 & 0 & 1 & 0 \end{array}\right] \quad -2R_1 + R_2 \Rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -5 & -2 & 0 & 1 \\ 0 & 0 & -3 & -2 & 1 & 1 \end{array}\right] \quad R_2 + R_3 \Rightarrow R_3$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & -5 & -2 & 0 & 1 \\ 0 & 0 & -3 & -2 & 1 & 1 \end{array}\right] \quad R_3 + R_1 \Rightarrow R_1 .$$

$$\left[\begin{array}{ccc|ccc} 2 & 1 & 0 & -1 & 1 & 1 \\ 0 & 1 & 0 & 4/3 & -5/3 & -2/3 \\ 0 & 0 & -3 & -2 & 1 & 1 \end{array}\right] \quad -5/3R_3 + R_2 \Rightarrow R_2$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & -7/3 & 8/3 & 5/3 \\ 0 & 1 & 0 & 4/3 & -5/3 & -2/3 \\ 0 & 0 & -3 & -2 & 1 & 1 \end{array}\right] \quad R_1 - R_2 \Rightarrow R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -7/6 & 8/6 & 5/6 \\ 0 & 1 & 0 & 4/3 & -5/3 & -2/3 \\ 0 & 0 & 1 & 2/3 & -1/3 & -1/3 \end{array}\right] \quad \begin{array}{l} 1/2R_1 \Rightarrow R_1 \\ -1/3R_3 \Rightarrow R_3 \end{array}$$

61. (A)

We have $(2, 4) = a(1,1) + b(2,3)$ so that

$$2 = a + 2b$$
$$4 = a + 3b$$

Thus $b = 2$ and $a = -2$ which implies

$$T(2,4) = -2T(1,1) + 2T(2,3) = -2(-1,1) + 2(1,2) = (4,2).$$

62. (B)

The function $f(z)$ is analytic if and only if the Cauchy-Riemann conditions are satisfied:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Thus

$$\frac{\partial v}{\partial y} = \frac{\partial u}{\partial x} = \cos x \cosh y$$

and

$$\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y} = -\sin x \sinh y\ .$$

We have

$$v(x,y) = \int \frac{\partial u}{\partial x}\, dy = \int \cos x \cosh y\ dy = \cos x \sinh y + g(x)\ .$$

Also,

$$-\sin x \sinh y = \frac{\partial v}{\partial x} = -\sin x \sinh y + g'(x)\ ,$$

so that $g(x) =$ constant. Hence $v(x,y) = \cos x \sinh y +$ constant.

63. (E)

$= T^{\prime}T = I_n, I_n$ being the identity transformation. The transformation $T^{\prime}$ is called the inverse of T and is denoted T^{-1}. Let $x, y \in E^n$ and assume $Tx = Ty$. Then $x = T^{-1}Tx = T^{-1}Ty = y$ so that T is one-to-one. If $Tx = 0$, then $x = 0$ since $T0 = 0$ and T is one-to-one. Thus the null space of $T = \{0\}$ and $\text{Dim } N(T) = 0$. The equation $n = \text{Dim } N(T) + \text{Dim } R(T)$ shows that $n = \text{Dim } R(T)$. Reversing the roles of T and T^{-1} above shows that T^{-1} is one-to-one; $N(T^{-1}) = \{0\}$, and $\text{Dim } N(T^{-1}) = 0$. Thus $\text{Dim } N(T^{-1}) \neq \text{Dim } R(T)$.

64. (D)

The given series is geometric, so its sum is $\dfrac{1}{1-\dfrac{2}{3}} = 3$.

65. (A)

Two circles C_1 and C_2 are said to be orthogonal if they interesect at right angles. This means that at a point of intersection $P(\bar{x},\bar{y})$ of C_1 and C_2, the radius r_1 of C_1 is tangent to C_2 at P and the radius of r_2 of C_2 is tangent to C_1 at P. Note that the centers of C_1 and C_2 are $(-a,-b)$ and $(-a',-b')$, respectively. The slope of the tangent line at the point of intersection $P(\bar{x},\bar{y})$ is equal to the negative reciprocal of the slope through $(-a,-b)$ and $P(\bar{x},\bar{y})$. It is also equal to the slope through the points $P(\bar{x},\bar{y})$ and $(-a',-b')$. Thus

$$\frac{-1}{\dfrac{\bar{y}+b}{\bar{x}+a}} = \frac{\bar{y}+b'}{\bar{x}+a'}$$

$$-[\bar{x}^2 + (a+a')\bar{x} + aa'] = \bar{y}^2 + (b+b')\bar{y} + bb'.$$

Multiplying by 2 and rearranging terms,

$$\Rightarrow -[(\bar{x}^2 + 2a\bar{x} + \bar{y}^2 + 2b\bar{y}) + (\bar{x}^2 + 2a'\bar{x} + \bar{y}^2 + 2b'\bar{y})]$$

$$= 2aa' + 2bb'c$$