LINEAR MODELS

ASSIGNMENT 9: DISCOUNTS

McDermott's Department Store is having a 20% discount sale on every item in the store.

1. a) What is the sale price of a $10 CD? _______________

   b) What is the sale price of a $50 calculator? _______________

2. On the graph paper provided, sketch a graph of the relationship between the retail price and the sale price of all items in the store. Let the horizontal axis be the actual retail price, and the vertical axis be the sale price. [HINT: Use the information found in answers 1a) and b) above as ordered pairs to start, and then try a few more prices before drawing the graph.

3. Describe the shape of the graph.

4. Use the graph to estimate the sale price of an item that retails at:

   a) $90 _______________

   b) $120 _______________

5. Use the graph to estimate the retail price of an item that is on sale for:

   a) $40 _______________

   b) $120 _______________

6. Construct an equation for this graph: let **r** be the retail price and **s** be the sale price.

7. Use the equation to find the exact values asked for in 4a), 4b) 5a) and 5b).

4a)___________ 4b)____________ 5a)______________ 5b)_________________

8. How do the exact values found in question 7 above compare with the estimated values found in questions 4 and 5?

9. If the graph of the equation is a line, what is the slope of this line?

____

10. What kind of information does the slope of this line give you for this problem? (How is the slope of the line related to how you would normally solve this problem?)

LINEAR MODELS

ASSIGNMENT 10: RYAN'S TEMPERATURE

Ryan, Denise's baby, looked a little under the weather, so Denise decided to take his temperature. Unfortunately the only thermometer Denise had available measured temperature in Centigrade (°C). She took Ryan's temperature with the thermometer anyway and found that he had a temperature of 38 °C. While Denise knew that a normal temperature on the Fahrenheit scale was 98.6 °F, she did not know whether 38 °C was a normal reading on the Centigrade scale. How should she interpret 38 °C ? Is this above normal, normal, or below normal?

Denise did remember from her chemistry class that the relationship between the Centigrade and Fahrenheit scales was linear. She also remembered from this class that the boiling point of water is 100 °C , but on the Fahrenheit scale it is 212 °F . Then Denise heard the weatherman say that it was going to be freezing tonight-- "the temperature will be 32 °F (which is also) or 0 °C ".

1. Use this information to sketch a graph (on the graph paper provided) of the relationship between the Fahrenheit and Centigrade scales.

2. Use the graph to decide whether Ryan's temperature is normal, above normal, or below normal.

3. Construct an equation for this relationship (Use C for Centigrade and F for Fahrenheit).

4. Use the equation to find Ryan's exact temperature in °Fahrenheit and determine if Ryan's temperature is normal, above, or below normal.

LINEAR MODELS

ASSIGNMENT 11: SLOPES AND EQUATIONS

1. Suppose that in a rectangular coordinate system the horizontal axis represents the number of items a company produces, and the vertical axis represents the profit, in dollars, the company earns on the production of that many items. How would you interpret the slope of the line segment joining the points (0, 0) and (100, 875)?

2. Suppose that in a rectangular coordinate system the horizontal axis labeled *t* represents the year, and the vertical axis labeled *P* represents the population of a small town at the end of year *t* . How would you interpret the slope of the line segment joining the points (1980, 89720) and (1990, 7456)?

3. Harry found that there was a linear relationship between the number of days he consistently studied math before a math test, and his scores on the math test. When he started studying consistently 6 days before the math test, he received a 62 on the test, on the other hand, when he started 10 days before the test, he received an 86 on the math test. Create a linear equation to describe the relationship between the number of days of consistent studying before the test and the test score. Use this equation to determine his score if he had started studying consistently 12 days before the test. How about if he saved his studying for the day before the test?

LINEAR MODELS

ASSIGNMENT 12

The following table contains employee data for a certain manufacturing company.

Years of Operation	x	1	2	3	4	5	6	7	8	9	10
Number of Employees	y	26	29	34	38	44	48	53	59	62	67

In this table, x represents the number of years the company has been operating, and y represents the number of employees working at the firm.

1. Plot this data on a set of coordinate axes.

2. Draw a straight line that gets "reasonably" close to as many data points as possible. (We call this the *line of best fit*.)

3. Estimate the slope of the line, and the y-intercept.

4. Given your estimated values of the slope and y-intercept, what is the equation of the line?

5. Check 4 data points with this equation.

6. Use the equation to predict how many employees will be with the company during the 12^{th} year.

LINEAR MODELS

ASSIGNMENT 13: THE JOB

Casey took a job at an appliance store in which he was given a weekly salary of $100 (base pay) plus a commission of 4% of his weekly gross sales.

1. On the graph paper provided, sketch a graph of the relationship between his salary and his gross sales, by letting the horizontal axis be his weekly gross sales, and the vertical axis be his weekly salary. [HINT: Pick a few possible weekly gross sales and then compute his corresponding salaries for those gross sales, and then draw your graph.]

2. Describe the shape of the graph.

3. Use the graph to estimate how much he would make if he has the following gross sales one week:

a) $2,000 _______________ b) $6,000 _______________

4. Use the graph to estimate what his weekly gross sales should be if he is to make $450 in a week.

____

5. Construct an equation for the base pay + commission plan: let **g** be his gross sales and **s** be his salary.

6. Use the equation to find the exact values asked for in questions 3a), 3b) and 4 above.

3a) __________ 3b) ____________ 4___________

7. How do the exact values found in question 6 compare with the estimated values found in questions 3 and 4?

8. If the graph of the equation is a line, what is the slope of this line?

____

9. What kind of information does the slope of this line give you for this problem? (How is the slope of the line related to how you would normally solve this problem?

About 6 months later Casey was given the option of continuing his base pay + commission salary described above, or taking a straight commission of 6% of the gross sales.

10. Sketch a graph of the relationship between his salary and his gross sales for the 6% pay plan on the same set of axes on which you graphed the salary + commission plan.

11. Use the graph to estimate how much he would make if he has the following gross sales one week:

a) $2,000 ______________ b) $6,000 ______________

12. Use the graph to estimate what his weekly gross sales should be if he is to make $450 in a week.

____

13. Construct an equation for the 6% commission pay plan: let **g** be his gross sales and **s** be his salary.

14. Use the equation to find the exact values asked for in questions 11a), 11b) and 12 above.

11a)________________ 11b)_________________ 12________________

15. How do the values found in question 14 for the straight commission compare with the values found in question 6 for the base pay + commission plan?

16. If the graph of this equation is a line, what is the slope of this line?

____

17. What kind of information does the slope of this line give you for this problem? (How is the slope of the line related to how you would normally solve this problem?

18. Look at the graphs of the two salary plans and write a page discussing under what conditions Casey should take the base pay + commission plan, and under what conditions he should take the straight commission plan. (And when it wouldn't matter which plan he chooses.) Back up your arguments with mathematics.

MATH 027: ASSIGNMENT 14

EXPONENTIAL GROWTH

1. Suppose Ana decides to save her money by putting \$2 in the bank on the first week, and then doubling the amount she saves each subsequent week (so she puts away \$4 the second week, \$8 the third week, etc.). Fill in the chart below to determine how much she puts in the bank on the 3rd, 4th, 5th, and 6th week and see if you can find a formula which describes how much money she puts in the bank on the nth week.

WEEK	Amount put in bank that week
1	\$2
2	\$4
3	
4	
5	
6	
n	

2. Use the formula to determine how much money she would save on the 20th week.

3. This type of growth is called **exponential growth**. On the graph paper provided, sketch a graph of the following three equations on the same graph: $y = x$, $y=x^2$, and $y=2^x$. What are the differences between the three graphs. Which increases faster.....what do we mean by "increases faster"?

MATH 027: IN-CLASS ASSIGNMENT 15

EXPONENTIAL GROWTH

1. A population of bacteria in a particular medium in a test tube doubles each hour. Suppose you start with a culture of 1000 bacteria in a test tube (at 0 hours). How many bacteria are present at the end of the first hour? at the end of the second hour?, the third hour? at the end of ***t*** hours? Fill in the chart below to determine the size of the population each hour and see if you can find a formula which describes the size of the population after ***t*** hours.

HOUR	POPULATION
1	2000=1000(2)
2	4000=1000(4)
3	
4	
5	
t	

2. Use the formula to determine how many bacteria will be in the test tube after 10 hours.

3. Letting the horizontal axis be the ***t*** axis (***t*** in hours) and the vertical axis be the ***P*** axis (size of population), sketch a graph of this equation and determine how long it would take for the population of bacteria to reach 10,000.

MATH 027: IN-CLASS ASSIGNMENT 16

EXPONENTIAL GROWTH

1. Suppose you put \$1000 in a bank which gives 5% simple interest per year. How much will you have in the bank at the end of the first year? ____________

at the end of the second year? _________________

at the end of the third year? _________________

at the end of the **sixth** year?___________________

2. Notice that for you to determine how much you would have at the end of the sixth year you would have to compute the principal (how much is in the bank) at the end of each year for the first five years. Let's see if we can use algebra to develop a formula to compute how much we will have without having to compute the intermediate end of year values for the previous years.

If you would put \$1000 in the bank at 5% simple interest, at the end of the year you would have

At the end of year 1:

$$A_1 = \underbrace{\mathbf{1000}}_{\text{Principal}} \overset{+}{+} \underbrace{\mathbf{1000}(.05)}_{\text{Interest on principal}}$$

which we can rewrite as **1000**(1+.05)=1000(1.05).

Why can we rewrite 1000 + 1000(.05) as 1000(1.05)?

Does it make sense to say that the new principal at the end of the first year is 105% that of the beginning of the year?

In general, the new principal at the end of the year is 105% of the old principal (the principal at the beginning of the year). Symbolically we can write:

A= 1.05P where **P** is the amount at the beginning of the year and **A** is the amount at the end of that year.

Let's examine what happens in the second year:

Our principal **starting in the second year** is now **1000(1.05)**, which is the principal for the end of the first year. To find the new amount in the bank at the *end of the second year* we take 105% of the amount in the bank at the *beginning of the second year* which is:

$$A_2 = \underbrace{\mathbf{1000(1.05)}}_{\text{Principal}} \underbrace{\times (1.05)}_{\times\ (1.05)}$$

which we can rewrite as $\mathbf{1000(1.05)^2}$.

Our principal **starting in the third year** is now **$1000(1.05)^2$**, which is the principal for the end of the second year. To find the new amount in the bank at the *end of the third year* we take 105% of the amount in the bank at the *beginning of the third year* which is:

$$\overset{\text{Principal}}{A_3 = \mathbf{1000(1.05)^2}} \overset{\text{X (1.05)}}{\text{ X (1.05)}} \text{ which we can rewrite as } \mathbf{1000(1.05)^3}.$$

Fill in the chart below and create the formula which describes how much (**A**) will be in the bank at the end of year **t**

(At the end of) year t	Amount (A) in the bank
1	$1000(1.05)$
2	$[1000(1.05)](1.05)=1000(1.05)^2$
3	$[1000(1.05)^2](1.05)=1000(1.05)^3$
4	
5	
t	

2. Use the formula to determine how much money will be in the bank at the end of 10 years.

3. How much money will be in the bank at the end of 25 years?

4. Suppose you put $1000 in a bank which pays 9% yearly simple interest. Develop a formula to compute how much will be in the bank at the end of t years.

5. Use the formula to find how much you will have in the bank after 10 years and after 20 years.

6. Suppose you put $12,000 in a treasury bond which yields 10% yearly interest provided you do not touch it for 15 years. How much will you have after the fifteen years?

7. Suppose you put \$10,000 in a bank which yields 8% simple yearly interest. Write an equation relating the amount (**A**) in the bank at the end of year **t**.

8. Letting the horizontal axis be the ***t*** axis (***t*** in years) and the vertical axis be the ***A*** axis (amount in the bank at the end of year **t**), sketch a graph of this equation and determine how long it would take for your money to double.

MATH 027: IN-CLASS ASSIGNMENT 17

EXPONENTIAL DECAY

1. In a learning experiment we taught a group of students 100 foreign words and then tested them and found that at the end of each day they would forget 1/2 of the words they still remembered. Fill in the chart below to determine how many words they still remembered at the end of the 1st, 2nd, 3rd, and 4th day and see if you can find a formula which describes how many words they remember at the end of the nth day.

END OF DAY	NUMBER OF WORDS REMEMBERED
1	$50=100(1/2)$
2	$25=50(1/2)=100(1/2)^2$
3	$12.5=25(1/2)=100(1/2)^3$
4	
n	

2. Use the formula to determine how many words the students would remember at the end of the 6th day.

3. Suppose that we found that in the same experiment (100 foreign words) we found that each day they would forget 10% of the words they still remembered. Fill in the chart below to determine how many words they still remembered at the end of the 1st, 2nd, 3rd, and 4th day and see if you can find a formula which describes how many words they remember at the end of the nth day.

END OF DAY	NUMBER OF WORDS REMEMBERED
1	
2	
3	
4	
n	

4. Use the formula to determine how many words the students would remember on the end of the 6th day.