**Problem statement**

a) The improper integral converges: \( \int_{0}^{\infty} xe^{-x^2} \, dx \). What is its value?

b) The value of \( \int_{0}^{\infty} e^{-x^2} \, dx \), another convergent improper integral, is \( \frac{\sqrt{\pi}}{2} \). This amazing fact is easiest to explain with some of the tools in third semester calculus. Improper integrals involving polynomials and \( e^{-x^2} \) often arise in statistics and therefore in analysis of experiments. Use integration by parts to get a formula relating \( \int_{0}^{\infty} x^n e^{-x^2} \, dx \) and \( \int_{0}^{\infty} x^{n-2} e^{-x^2} \, dx \), where \( n \) is a positive integer bigger than 2. (The parts to take are slightly tricky.)

c) Now find the values of

i) \( \int_{0}^{\infty} x^2 e^{-x^2} \, dx \)

ii) \( \int_{0}^{\infty} x^3 e^{-x^2} \, dx \)

iii) \( \int_{0}^{\infty} x^4 e^{-x^2} \, dx \)

You will need the reduction formula in b) and the two initial values found in a) and b).