Problem statement A graph of the derivative of $f(x)$ is displayed below. Information about the function $f(x)$ is known only for $-2.5 < x < 3.5$. Also $f(-2) = 1$. Consider the graph carefully, and consider the information in both the numbers and the shapes of the graph (both “quantitative” and “qualitative” information)!

![Graph of $y = f'(x)$, the derivative of $f(x)$](image)

a) Explain why $-2 < f(0) < -1$. Look carefully at the graph and make estimates using the MVT. Explain the steps of your reasoning in detail.

b) Explain why $f(3) > 4 + f(1)$. Again, use the MVT and explain your reasoning in detail.

c) How big and how small can $f(1) - f(0)$ be?

d) Use the information in a), b), and c) to explain why $f(3)$ must be positive.

e) Explain why $f(x) = 0$ must have a solution between 0 and 3. Use the IVT and the information obtained in previous parts of this problem.