**Problem statement** Polynomials have roots. For example, \((x - 1)(x + 2)(x - 3) = x^3 - 2x^2 - 5x + 6\) which means that \(x^3 - 2x^2 - 5x + 6 = 0\) when \(x = 1\) or \(x = -2\) or \(x = 3\). Low degree polynomials have algebraic recipes for roots in terms of their coefficients. There are no such formulas for higher degree polynomials. If \(r_1, r_2,\) and \(r_3\) are the roots of a third degree polynomial, \(x^3 + Ax^2 + Bx + C\), then the coefficients (\(A, B,\) and \(C\)) are functions of the roots (\(r_1, r_2,\) and \(r_3\)).

a) What are the functions? That is, write \(A, B,\) and \(C\) as functions of \(r_1, r_2,\) and \(r_3\).

Verify if \(\begin{cases} r_1 = 1 \\ r_2 = -2 \\ r_3 = 3 \end{cases}\) then \(\begin{cases} A = -2 \\ B = -5 \\ C = 6 \end{cases}\).

b) Suppose the roots are changed: \(\begin{cases} r_1 : 1 \to 1.02 \\ r_2 : -2 \to -2.04 \\ r_3 : 3 \to 2.95 \end{cases}\). Use partial derivatives and linearization to predict the approximate changes in the coefficients.

c) Suppose now that the coefficients are changed. That is, consider new coefficients: \(\begin{cases} A = -2.03 \\ B = -5.02 \\ C = 6.01 \end{cases}\). Approximate the roots which would give these coefficients. (This is harder, and a new idea is needed: what perturbations in the roots will, to first order, give these perturbations in the coefficients? A “system” of three linear equations in three unknowns must be solved.)