

Problem statement The following problem is quoted directly from D. Knuth, *The Art of Computer Programming*, the standard (multivolume) reference for theoretical computer science. This problem is in volume 1, *Fundamental Algorithms*.

If $f(x) = \sum_{k \geq 0} a_k x^k$, and this series converges for $x = x_0$, then show that

$$\sum_{k \geq 0} a_k x_0^k H_k = \int_0^1 \frac{f(x_0) - f(x_0 y)}{1 - y} dy.$$

Here the numbers H_k are defined to be the partial sums of the harmonic series: $H_0 = 0$; $H_k = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{k}$ for $k \geq 1$.

Hint Expand everything on the right side in a power series in x_0 . What is the coefficient of x_0^k ? The formula for the sum of a *finite* geometric series will be useful.