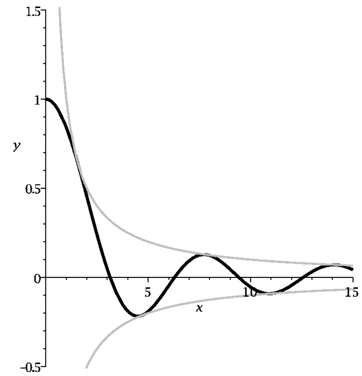
Problem statement The function $\dfrac{\sin x}{x}$ can be extended to have value 0 when $x = 0$ (for example, using l'Hôpital's Rule). This function occurs in many applications, such as signal processing. A graph is shown to the right. The "bumps" when $x > 0$ touch the displayed curves $y = \pm\dfrac{1}{x}$. When x gets large, the area under the bumps, positive and negative, almost cancels. The quantity $\displaystyle\int_0^\infty \frac{\sin x}{x}\,dx$ is finite. Here is one way to find the exact value of this integral.



a) Suppose $f(t) = \displaystyle\int_0^\infty \left(\frac{\sin x}{x}\right) e^{-tx}\,dx$. Compute $f'(t)$, the derivative of f with respect to t. The resulting integral can be evaluated using integration by parts, and you should conclude that $f'(t) = -\dfrac{1}{1+t^2}$.

b) Solve the differential equation $f'(t) = -\dfrac{1}{1+t^2}$. If $t \to +\infty$, the value of the integral defining $f(t)$ approaches 0. Then the general solution of the differential equation which involves an arbitrary additive constant can be used to get an exact formula for (t).

c) So $f(0)$ is $\displaystyle\int_0^\infty \left(\frac{\sin x}{x}\right) e^{-\mathbf{0}x}\,dx = \int_0^\infty \frac{\sin x}{x}\,dx$ which can be evaluated using b).