

Problem statement A solid body of uniform density ρ fills the portion of space lying in the first octant and between two spheres centered at the origin of radius a and b , respectively, with $a < b$. The coordinates of points in this solid are those which satisfy $x, y, z \geq 0$ and $a^2 \leq x^2 + y^2 + z^2 \leq b^2$.

a) Use integration in spherical coordinates to find the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the center of mass of this body. (How can symmetry simplify this problem?)

b) Find the coordinates of the center of mass when the body becomes a very thin spherical shell—that is, find the limiting value of the position of the center of mass from part a) as the inner radius a approaches the outer radius b .