Correction In problem A, the phrase “$-iR$ to $R$” has been changed to “$R - iR$ to $R$” which is what I meant and what the accompanying diagram now indicates more definitely. I thank Mr. Marcus for this correction (2/17).

Due at the beginning of class, Wednesday, February 24, 2010

Solve these textbook problems (4 points each):

1.6 (page 73): 1, 4 (see page 60), 7 (there’s an answer in the back of the book but please try to get a different verification using Green’s Theorem).

2.2 (page 103): 2, 9, 10. (This section has problems about power series. We have not covered most of the material in it yet. Problem 2 asks for the radius of convergence. This follows directly from things done in class. For problems 9 and 10, get the requested series using series which are already known and simple “tricks” first used in calc 2.)

A. (10 points) Suppose $R$ is a large positive real number, and $Q_R$ is the line segment $S_R$ from $R - iR$ to $R$ followed by the quarter circular arc $C_R$ from $R$ to $iR$ as shown. If $I_R = \int_{Q_R} e^{-z} \frac{z^2}{z^2 - 4z + 15} \, dz$, find an estimate for $|I_R|$ and use the estimate to verify that $\lim_{R \to \infty} I_R = 0$.

B. (10 points) Verify Green’s Theorem for the region $R$ in the plane whose boundary $C$ is the parabolic arc $y = 1 - x^2$ (for $-1 \leq x \leq 1$) and the line segment $y = 0$. Assume that the boundary has the correct orientation, and that $P(x, y)$ and $Q(x, y)$ have continuous partial derivatives on the boundary curves and at all points of the interior open set. Use the 1 variable Fundamental Theorem of Calculus to prove $\int_C P(x, y) \, dx + Q(x, y) \, dy = \int_R \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dx \, dy$. 