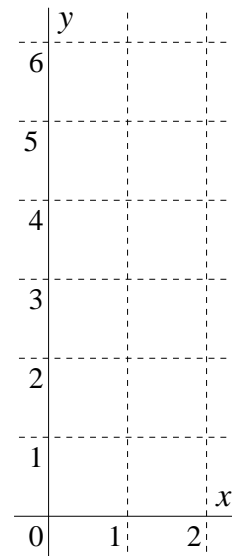


- (14) 1. This problem investigates the iterated integral  $I = \int_0^2 \int_0^{6-x^2} x \, dy \, dx$ .
- a) Compute  $I$ .
- b) Use the axes to the right to sketch the region of integration for  $I$ .
- c) Write  $I$  as a sum of one or more  $dx \, dy$  integrals. You do not need to compute the result!



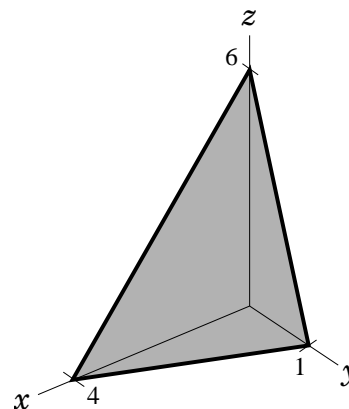
- (10) 2. Find and classify using the Second Derivative Test all critical points of  $f(x, y) = x^2 - y + \ln(3x + y)$ .

Resembling problem 19 in section 14.7

- (12) 3. Use the Lagrange multiplier method to find the maximum and minimum values of  $f(x, y, z) = 3x - y + 2z$  subject to the constraint  $x^2 + y^2 + z^2 = 1$ .

Similar to a problem done in class and several homework problems

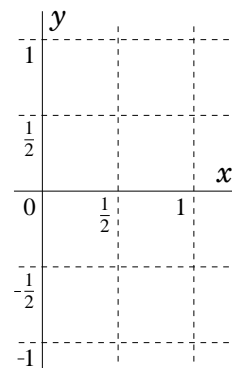
- (16) 4. Calculate the triple integral of  $f(x, y, z) = y$  over the tetrahedron  $W$  shown to the right.



Resembling but much easier than problem 17 in section 15.3

- (14) 5. Sketch the  $D$  indicated on the axes provided and integrate  $f(x, y)$  over  $D$  using polar coordinates.
- $$f(x, y) = x(x^2 + y^2)^{-1}; \quad x \geq \frac{1}{2}, \quad x^2 + y^2 \leq 1$$

Resembling problem 5 in section 15.4



- (12) 6. Use cylindrical coordinates to find the mass of a cylinder of radius 4 and height 7 if the mass density at a point is equal to the square of the distance from the cylinder's central axis.

Resembling problem 42 in section 15.4

- (12) 7. A three dimensional region  $R$  is those points in the first octant with distance to the origin between 2 and 3. Compute the triple integral of  $z^2$  over this region.

**Suggestion** Spherical.

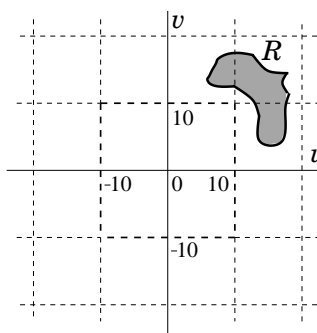
- (10) 8. This problem is about the transformation  $F$  defined by  $\begin{cases} x = u^2 - v^2 \\ y = 2uv \end{cases}$ .

a) Compute the Jacobian of this transformation.

b) Suppose  $R$  is in the  $u, v$  plane and that  $Q$  is the image under  $F$  of  $R$  in the  $x, y$  plane.

The area of  $Q$  is  $\iint_Q 1 dA_{x,y}$ . Use the Change of Variables Formula to write an integral expression which is equal to this area using the  $u, v$  variables over  $R$  and the specific Jacobian computed in a).

c) A region  $R$  is shown in the  $u, v$  plane. Look carefully at the location of  $R$  compared to the coordinate axes. The region is mapped to a region  $Q$  in the  $x, y$  plane by the mapping whose Jacobian is computed in part a). If the area of  $R$  is at least 50, use complete English sentences to explain why the area of the image region  $Q$  is at least 40,000.



**Hint** No exact computation can be made since precise information isn't given. Underestimate the Jacobian for  $(u, v)$  in  $R$  using  $R$ 's location. Combine this with b)'s answer to get the desired result.

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**Second Exam for Math 251, sections 12–17**

November 16, 2010

NAME \_\_\_\_\_

SECTION \_\_\_\_\_

**Do all problems, in any order.****Show your work. An answer alone may not receive full credit.****No notes and no calculators may be used on this exam.**

**“Simplification” of answers is not necessary,  
but standard values of traditional functions  
such as  $e^0$  and  $\sin(\frac{\pi}{2})$  should be given.**

Problem Number	Possible Points	Points Earned:
1	14	
2	10	
3	12	
4	16	
5	14	
6	12	
7	12	
8	10	
Total Points Earned:		

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