1. This problem investigates the iterated integral \( I = \int_0^2 \int_0^{6-x^2} x \, dy \, dx \).
   a) Compute \( I \).
   b) Use the axes to the right to sketch the region of integration for \( I \).
   c) Write \( I \) as a sum of one or more \( dx \, dy \) integrals. You do not need to compute the result!

2. Find and classify using the Second Derivative Test all critical points of \( f(x, y) = x^2 - y + \ln(3x + y) \).
   Resembling problem 19 in section 14.7

3. Use the Lagrange multiplier method to find the maximum and minimum values of \( f(x, y, z) = 3x - y + 2z \) subject to the constraint \( x^2 + y^2 + z^2 = 1 \).
   Similar to a problem done in class and several homework problems

4. Calculate the triple integral of \( f(x, y, z) = y \) over the tetrahedron \( W \) shown to the right.
   Resembling but much easier than problem 17 in section 15.3

5. Sketch the \( D \) indicated on the axes provided and integrate \( f(x, y) \) over \( D \) using polar coordinates.
   \( f(x, y) = x(x^2 + y^2)^{-1}; \quad x \geq \frac{1}{2}, \quad x^2 + y^2 \leq 1 \)
   Resembling problem 5 in section 15.4
(12) 6. Use cylindrical coordinates to find the mass of a cylinder of radius 4 and height 7 if the mass density at a point is equal to the square of the distance from the cylinder’s central axis.

Resembling problem 42 in section 15.4

(12) 7. A three dimensional region $R$ is those points in the first octant with distance to the origin between 2 and 3. Compute the triple integral of $z^2$ over this region.

**Suggestion** Spherical.

(10) 8. This problem is about the transformation $F$ defined by
\[
\begin{align*}
    x &= u^2 - v^2 \\
    y &= 2uv
\end{align*}
\]

a) Compute the Jacobian of this transformation.
b) Suppose $R$ is in the $u,v$ plane and that $Q$ is the image under $F$ of $R$ in the $x,y$ plane. The area of $Q$ is $\int \int_Q 1 \, dA_{x,y}$. Use the Change of Variables Formula to write an integral expression which is equal to this area using the $u,v$ variables over $R$ and the specific Jacobian computed in a).
c) A region $R$ is shown in the $u,v$ plane. Look carefully at the location of $R$ compared to the coordinate axes. The region is mapped to a region $Q$ in the $x,y$ plane by the mapping whose Jacobian is computed in part a). If the area of $R$ is at least 50, use complete English sentences to explain why the area of the image region $Q$ is at least 40,000.

**Hint** No exact computation can be made since precise information isn’t given. Underestimate the Jacobian for $(u,v)$ in $R$ using $R$’s location. Combine this with b)’s answer to get the desired result.
Second Exam for Math 251, sections 12–17

November 16, 2010

NAME ____________________________________________

SECTION ______

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes and no calculators may be used on this exam.

“Simplification” of answers is not necessary, but standard values of traditional functions such as $e^0$ and $\sin\left(\frac{\pi}{2}\right)$ should be given.

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