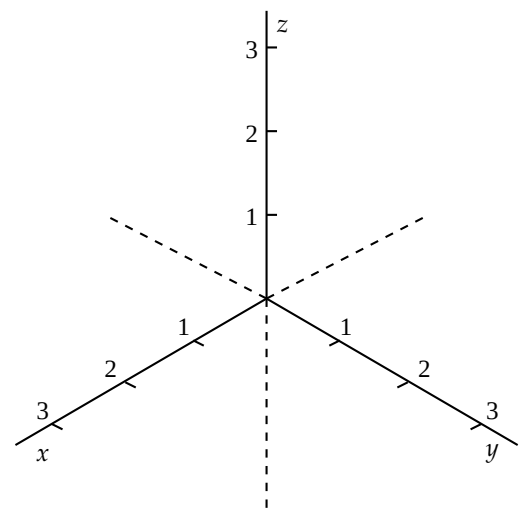
(16) 1. In this problem, $f(x,y,z) = 3\sqrt{2yz - 3x^2}$ and p is the point $(1,2,3)$ so that $f(p) = 9$.
a) Compute $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$, and $\frac{\partial f}{\partial z}$. Then find the value of ∇f at p. (The result will be a vector of integers.)
b) Find an equation for the plane tangent to $f(x,y,z) = 9$ at p. You need *not* simplify your answer!
c) Find a unit vector in the direction of the largest directional derivative of f at p. You need *not* simplify your answer! Also find the value of this directional derivative. You need *not* simplify your answer!
d) Find *one* unit vector so that the directional derivative of f at p in the direction of this vector is 0. (The magnitude of the vector should be 1!)

(20) 2. Suppose $Q(x,y)$ is defined by the equation $Q(x,y) = e^{xf(y)}$ where f is a differentiable function of one variable with $f(0) = A$, $f'(0) = B$, and $f''(0) = C$. Use this information to compute these quantities (your answers may involve constants and A, B, and C):
a) $Q(0,0)$ b) $\frac{\partial Q}{\partial x}(0,0)$. c) $\frac{\partial Q}{\partial y}(0,0)$. d) $\frac{\partial^2 Q}{\partial x^2}(0,0)$. e) $\frac{\partial^2 Q}{\partial x \partial y}(0,0)$. f) $\frac{\partial^2 Q}{\partial y^2}(0,0)$.

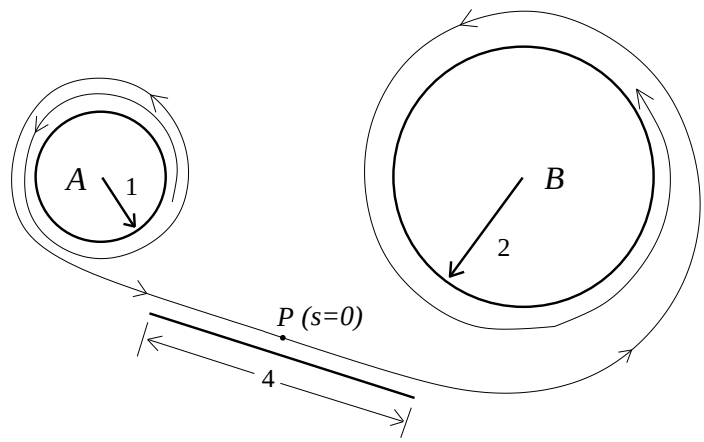
(16) 3. Suppose L is the line containing the point $(3,0,1)$ and the <u>center of the sphere</u> S defined by $x^2+y^2+z^2 = 5$.
a) Find parametric equations for L.
b) The line L and the sphere S intersect in two points, p and q. Find the coordinates of these points. You need *not* simplify your answers!
c) What is the distance between p and q?
d) Sketch the sphere S, the line L, and the points p and q as well as you can on the axes to the right. Be sure that p and q are labeled.



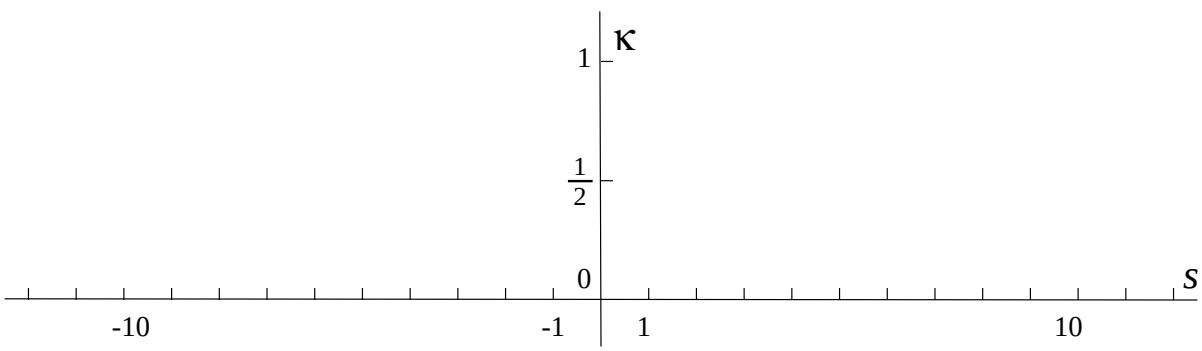
(18) 4. Suppose the position vector of a moving point is given by $\mathbf{r}(t) = \langle 3\sin t, e^{-2t}, t^2 \rangle$.
a) Find the velocity vector of the point.
b) Write an integral expression for the length of the curve from $t = -1$ to $t = 2$. Do *not* "simplify" and do *not* compute the value of the integral.
c) Find the acceleration vector of the point.
d) Find $\vec{v}$, the velocity vector at $t = 0$, and $\vec{a}$, the acceleration vector at $t = 0$.
e) Write $\vec{a}$ as a sum of two vectors, $\vec{a}_{\parallel}$ and $\vec{a}_{\perp}$, so that $\vec{a}_{\parallel}$ is tangent to the path of the particle when $t = 0$ and $\vec{a}_{\perp}$ is perpendicular to the path of the particle when $t = 0$. (So $\vec{a}_{\parallel}$ is a scalar multiple of v and $\vec{a}_{\perp}$ is perpendicular to v.)

(8) 5. Suppose $p = (1,0,2)$, $q = (0,2,2)$, and $r = (1,1,1)$ are points in $\mathbb{R}^3$.
a) Find a vector orthogonal to the plane through the points p, q, and r.
b) Find the area of triangle pqr.

(14) 6. A point is moving along the curve below in the direction indicated. Its motion is parameterized by arc length, s, so that it is moving at unit speed. Arc length is measured from the point P (both backward and forward). The curve is intended to continue indefinitely both forward and backward in s, with its forward motion curling more and more tightly around the indicated circle, B, and, backward, curling more and more tightly around the other circle, A. Near P the curve is parallel to the indicated line segment.



Sketch a graph of the curvature, κ, as a function of the arc length, s. What are $\lim_{s\to+\infty} \kappa(s)$ and $\lim_{s\to-\infty} \kappa(s)$? Use complete English sentences to briefly <u>explain</u> the numbers you give. Also, please <u>explain</u> the behavior of the graph near $s = 0$.



Note that the units on the horizontal and vertical axes differ in length.

<u>Explanations</u> (three are requested: one for each limit, and one for behavior near 0):

(8) 7. Suppose y is implicitly defined as a function of x and z by the equation

$$x^2y + 3xy^2z - 4z^3x = 5\,.$$

Find a formula for $\frac{\partial y}{\partial z}$.

First Exam for Math 251, sections 12–17

October 12, 2010

NAME _____

SECTION _____

Do all problems, in any order.

Show your work. An answer alone may not receive full credit.

No notes and no calculators may be used on this exam.

"Simplification" of answers is not necessary, but standard values of traditional functions such as e^0 and $\sin\left(\frac{\pi}{2}\right)$ should be given.

Problem Number	Possible Points	Points Earned:
1	16	
2	20	
3	16	
4	18	
5	8	
6	14	
7	8	
Total Points Earned:		