8a. z is defined implicitly as a function of x & y by the equation $z^2y-3xy^2+xz=3$. Find $\frac{dz}{dx}$ at the point $p=(1,2,3)$.

$$\frac{dz}{dx}=-\frac{F_x}{F_z}=\frac{-(-3y^2)+z}{2yz+x}=\frac{dz}{dx}=\frac{3y^2-z}{2yz+x}. \quad z_x(P)=z_x(1,2,3)=\frac{9}{13}$$

b. $f(x,y,z)=z^2y-3xy^2+xz$. If $p=(1,2,3)$, find the maximum rate of change of f at p & find a vector in the direction of this maximum increase.

$$\nabla F=\langle F_x,F_y,F_z\rangle=\langle -3y^2+z,\ z^2-6xy,\ 2yz+x\rangle$$

$$\nabla_p F=\nabla F(1,2,3)=\langle -9,-3,13\rangle$$

$$\|\nabla_p F\|=\sqrt{(-9)^2+(-3)^2+13^2}=\sqrt{81+9+169}=\sqrt{259}=\text{max. rate of change.}$$

unit vector in the direction of this max. increase $=\frac{\nabla_p F}{\|\nabla_p F\|}=\left\langle \frac{-9}{\sqrt{259}},\frac{-3}{\sqrt{259}},\frac{13}{\sqrt{259}}\right\rangle$

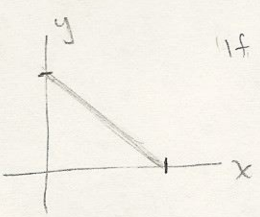
c. Find an equation for the plane tangent to $f(x,y,z)=z^2y-3xy^2+xz$ at ~~that~~ the point $P=(1,2,3)$.

$$\nabla_p F=\langle -9,-3,13\rangle$$

$$-9(x-1)-3(y-2)+13(z-3)=0$$

9. Bounded region in the first octant of $\mathbb{R}^3$ has the surface defined by $x+y+z^2=1$ as part of its boundary. The remainder of its boundary is given by portions of the planes $x=0$, $y=0$, and $z=0$. Compute the triple integral of z over this region in space.

$$\int_0^1\int_0^{1-x}\int_0^{\sqrt{1-x-y}} z\,dz\,dy\,dx$$



If $z=0$, $x+y=1$

$y=1-x$

$x: 0$ to 1

$y: 0$ to $1-x$

$z: 0$ to $\sqrt{1-x-y}$

$$\left.\frac{z^2}{2}\right|_0^{\sqrt{1-x-y}}$$

$$\frac{1}{2}\int_0^1\int_0^{1-x} 1-x-y\,dy\,dx$$

$$\frac{1}{2}\int_0^1\left(y-xy-\left.\frac{y^2}{2}\right|_0^{1-x}\right)dx=\frac{1}{2}\int_0^1\left(1-x-x(1-x)-\frac{(1-x)^2}{2}\right)dx=\frac{1}{2}\int_0^1(1-2x+x^2)dx-\frac{1}{4}\int_0^1(1-x)^2dx$$

$$=\frac{1}{2}\left(x-x^2+\left.\frac{x^3}{3}\right|_0^1\right)-\frac{1}{4}\int_0^1 1-2x+x^2\,dx$$

$$=\frac{1}{2}\left(1-1+\frac{1}{3}\right)-\frac{1}{4}\left(x-x^2+\left.\frac{x^3}{3}\right|_0^1\right)$$

$$\frac{1}{6}\quad-\frac{1}{4}\left(1-1+\frac{1}{3}\right)$$

$$\frac{1}{6}-\frac{1}{4}\left(\frac{1}{3}\right)=\frac{1}{12}$$