7) Suppose $F(x,y,z) = (x + 3xz^2 + e^{zy^5} + x^3)i + (y^3 - 17z^9)j + (\arctan(xy^4) + z^2)k$, a vector field defined and continuously differentiable throughout $\mathbb{R}^3$. Compute the total flux outward of F through the sphere of radius 2 centered at $(0,0,0)$. Some methods are easier than others. Be careful with your initial computations because the formula for F has eight "pieces"!

Think Divergence theorem!

$P = (x + 3xz^2 + e^{zy^5} + x^3$ $\quad Q = y^3 - 17z^9 \quad R = \arctan(xy^4) + z^2$

$P_x = (1 + 3z^2 + 3x^2)\,dx \quad Q_y = 3y^2\,dy \quad R_z = 2z\,dz$

$$= \iiint \overbrace{(1 + 3z^2 + 3x^2 + 3y^2 + 2z)}^{P_x + Q_y + R_z =}\,dV$$

Think spherical coordinates!

$dV = \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

$$= \iiint (1 + 3z^2 + 3x^2 + 3y^2 + 2z)\,dV$$

$$= \iiint (1 + 3\underbrace{(z^2 + x^2 + y^2)}_{(\rho^2)} + \underbrace{2z}_{(\rho\cos\phi)})\,dV$$

$x = \rho\cos\theta\sin\phi$

$y = \rho\sin\theta\sin\phi$

$z = \rho\cos\phi$

$0 \le \rho \le 2$

$0 \le \phi \le \pi$

$0 \le \theta \le 2\pi$

$$= \int_0^{\theta=2\pi}\int_0^{\phi=\pi}\int_{\rho=0}^{2} (1 + 3\rho^2 + 2\rho\cos\phi)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \int_0^{\theta=2\pi}\int_0^{\phi=\pi}\int_{\rho=0}^{2} \rho^2 \sin\phi + 3\rho^4 \sin\phi + 2\rho^3 \sin\phi\cos\phi \, d\rho \, d\phi \, d\theta$$

$$\tfrac{1}{3}\rho^3 \sin\phi + \tfrac{3}{5}\rho^5 \sin\phi + \tfrac{1}{2}\rho^4 \sin\phi\cos\phi \Big|_0^2$$

$$\int_0^{2\pi}\int_0^{\pi} \left(\tfrac{8}{3}\sin\phi + \tfrac{96}{5}\sin\phi + 8\sin\phi\cos\phi\right) d\phi \, d\theta$$

$$\int_0^{2\pi}\left(\int_0^{\pi}\left(\tfrac{328}{15}\sin\phi\right) d\phi + \int_0^{\pi}(8\sin\phi\cos\phi)\,d\phi\right) d\theta$$

let $u = \sin\phi$ $\quad \int \tfrac{32}{3} u\,du = \tfrac{32}{6}u^2\,du$

$du = \cos\phi$

$$\int_0^{2\pi}\left(-\tfrac{328}{15}\cos\phi \Big|_0^{\pi} + 8(\sin\phi)^2 \Big|_0^{\pi}\right) d\theta$$

$$\int_0^{2\pi}\left(-\tfrac{328}{15}(-1) - \left[-\tfrac{328}{15}(1)\right] + 0\right) d\theta$$

$$\int_0^{2\pi} \tfrac{656}{15}\,d\theta = \tfrac{(656)2\pi}{15} - 0 = \boxed{\tfrac{1312\pi}{15}}$$