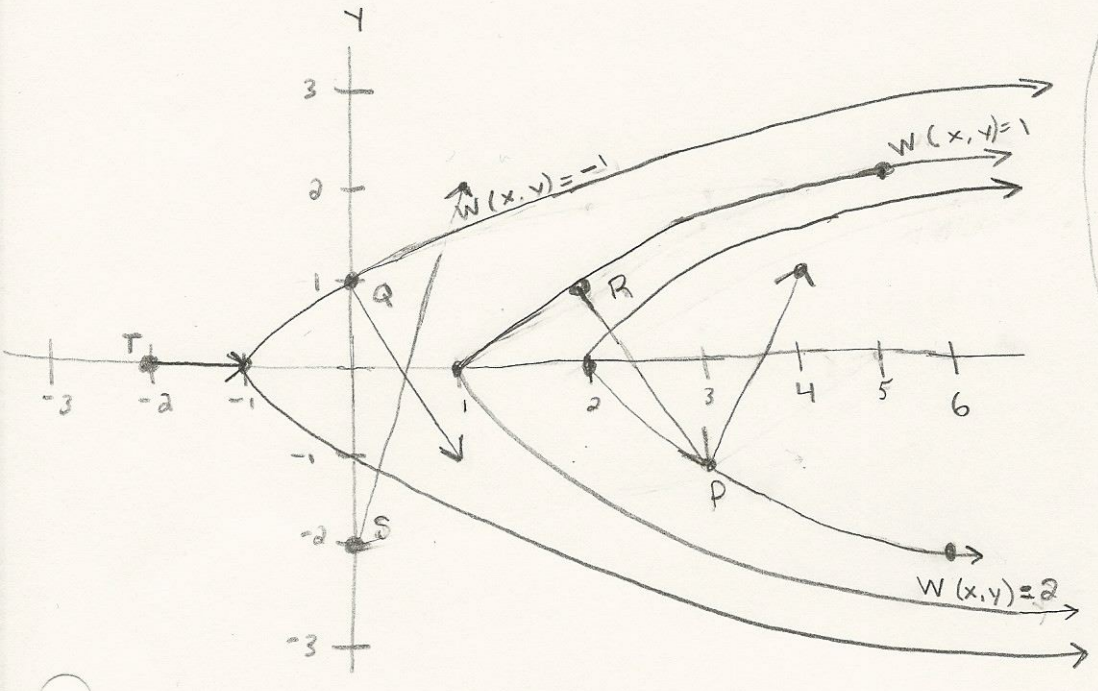
6) Sketch the three level curves of the function $W(x,y) = x - y^2$ which pass through the points $P=(3,-1)$ and $Q=(0,1)$ and $R=(2,1)$. Be sure to label each curve with the appropriate function value and be sure that your drawing is clear and unambiguous.

Also sketch on the same axis the vectors of the gradient vector field ∇W at the points P and Q and R and S and T. The point $S=(0,-2)$ and $T=(-2,0)$.



$\nabla W(0,-2) = \langle 1, 4 \rangle$ @ S

$\nabla W(-2,0) = \langle 1, 0 \rangle$ @ T

$W(x,y) = x - y^2$

$W(P) = 3 - (-1)^2 = 2 \rightarrow 2 = x - y^2 \rightarrow -2 + x = y^2 \rightarrow \pm\sqrt{-2+x} = y$ $(3,-1)$

$W(Q) = 0 - (1)^2 = -1 \rightarrow -1 = x - y^2 \rightarrow 1 + x = y^2 \rightarrow \pm\sqrt{1+x} = y$ $(0,1)$

$W(R) = 2 - (1)^2 = 1 \rightarrow 1 = x - y^2 \rightarrow -1 + x = y^2 \rightarrow \pm\sqrt{-1+x} = y$ $(2,1)$

$\nabla W = \langle 1, -2y \rangle$

$\nabla W(3,-1) = \langle 1, 2 \rangle$ @ P

$\nabla W(0,1) = \langle 1, -2 \rangle$ @ Q

$\nabla W(2,1) = \langle 1, -2 \rangle$ @ R

$Y = -\sqrt{-2+x}$

X	Y
2	0
3	-1
6	-2

$Y = \sqrt{1+x}$

X	Y
0	1
2	$\sqrt{3} \approx 1.73$
3	2
-1	0

$Y = \sqrt{-1+x}$

X	Y
1	0
3	$\sqrt{2} \approx 1.41$
5	2
2	1