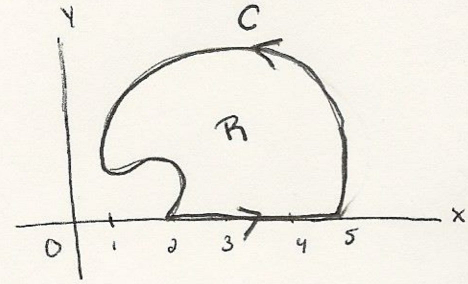
5) A region R in $\mathbb{R}^2$ is located in the first quadrant, as shown. Its boundary, oriented counter counterclockwise as shown, is an interval $I = [2,5]$ on the x-axis + a curve C in the first quadrant.



Suppose the following information is also known:

$$\iint_R 1\,dA = 5; \quad \iint_R x\,dA = 12; \quad \iint_R y\,dA = 8.$$

Find $\int_C (x^2+xy+3y)\,dx + (\arctan(y^3) + 3x^2 + 2xy + x)\,dy$.

Think Green's Theorem

$$\int_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$P(x,y) = x^2 + xy + 3y$ $\qquad$ $Q(x,y) = \arctan(y^3) + 3x^2 + 2xy + x$

$\frac{\partial P}{\partial y} = x + 3$ $\qquad$ $\frac{\partial Q}{\partial x} = 6x + 2y + 1$

$$\iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$\iint_R (6x + 2y + 1 - x - 3)\,dA$$

$$\iint_R (5x + 2y - 2)\,dA$$

$$= 5\underbrace{\iint_R (x)\,dA}_{} + 2\underbrace{\iint_R (y)\,dA}_{} + (-2)\underbrace{\iint_R (1)\cdot dA}_{}$$

$$= 5(12) + 2(8) + (-2)(5) = 66$$

$$\int_C (x^2 + xy + 3y)\,dx + (\arctan(y^3) + 3x^2 + 2xy + x)\,dy = 66.$$