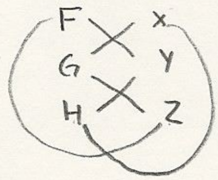
3) Suppose $F(x,y,z) = \overset{F}{(4y^5e^{4x} + 3z)}\,i + \overset{G}{(5y^4e^{4x})}\,j + \overset{H}{(3x + 2z)}\,k$, a vector field defined and continuously differentiable throughout $\mathbb{R}^3$.

a) Determine whether there is a scalar function $P(x,y,z)$ defined everywhere in $\mathbb{R}^3$. [If there is] such a P, find it; if there is not, explain why not.



$F_y = 20y^4e^{4x}$ ☑ $\quad G_z = 0$ ☑

$G_x = 20y^4e^{4x} \quad H_y = 0$

$F_z = 3$ ☑

$H_x = 3$

Theres a potential!

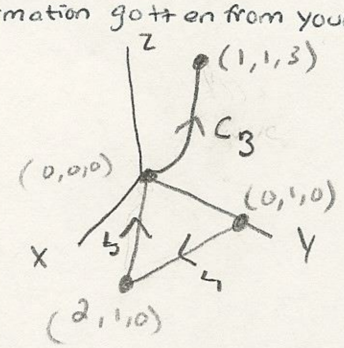
$\int F\,dx = y^5e^{4x} + 3zx$ + any function of z and y

$\int G\,dy = y^5e^{4x}$ + any function of x and z

$\int H\,dz = 3xz + z^2$ + any function of x and y

$$\boxed{P(x,y,z) = y^5e^{4x} + 3zx + z^2}$$

b) Compute the integral $\int_C F\cdot ds$, where C is the path shown to the right. It is a line segment from $(0,1,0)$ to $(2,1,0)$ followed by a line segment from $(2,1,0)$ to $(0,0,0)$ followed by the parametric curve $\langle t, t, 3t^2\rangle$ for $0 \le t \le 1$ which connects $(0,0,0)$ to $(1,1,3)$. Use information gotten from your answer to a) to help if you wish. Some methods are easier than others.



$\int_C F\cdot ds = \underbrace{\varphi(Q)}_{\text{terminal point}} - \underbrace{\varphi(P)}_{\text{initial point}}$ * used if there is a potential.

$\underbrace{P(0,1,0)}_{P} = (1)^5e^{4(0)} + 3(0)(0) + (0)^2 = 1$

$\underbrace{P(1,1,3)}_{Q} = (1)^5e^{4(1)} + 3(3)(1) + (3)^2 = e^4 + 18$

$\int_C F\cdot ds = (e^4 + 18) - (1)$

$\int_C F\cdot ds = e^4 + 17$

CLEARLY!