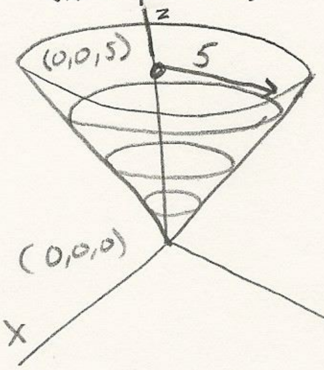
2) Suppose C is the solid right circular cone shown to the right. Its height is 5 units, with vertex at the origin, $(0,0,0)$, and the base is centered at $(0,0,5)$ with radius equal to 5. Compute the triple integral of

$$\frac{z}{(x^2+y^2+1)^3}$$ over the solid object.

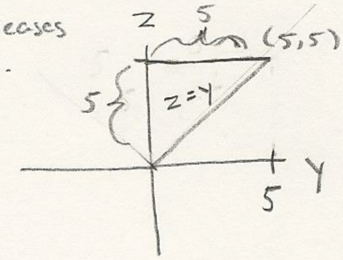


Cylindrical Coordinates: $dA = r\,dr\,d\theta\,dz$

$r: 0 \text{ to } z$ } r (radius) increases as z increases.

$\theta: 0 \text{ to } 2\pi$

$z: 0 \text{ to } 5$



$$f = \frac{z}{(\underbrace{x^2+y^2}_{r^2}+1)^3} \quad \int_0^{\theta=2\pi}\int_0^{z=5}\int_0^{r=z} \frac{z}{(r^2+1)^3}\, r\,dr\,dz\,d\theta$$

Let $u = (r^2+1)$

$du = 2r$

$$= \int \frac{z}{2} u^{-3}\,du = \frac{-z}{4} u^{-2} = \frac{-z}{4(r^2+1)^2}\Big|_0^z = \frac{-z}{4(z^2+1)^2} + \frac{z}{4}$$

$$\int_0^{2\pi}\left(\int_0^5 \frac{-z}{4(z^2+1)^2}\,dz + \int_0^5 \frac{z}{4}\,dz\right)d\theta$$

let $u = z^2+1$

$du = 2z$

$$\frac{-1}{8}u^{-2} = \frac{1}{8}u^{-1} = \frac{1}{8(z^2+1)}\Big|_0^5 = \frac{1}{8(26)} - \frac{1}{8}$$

$$\frac{1}{8}z^2\Big|_0^5 = \frac{1}{8}(25)$$

$$\int_0^{2\pi}\left(\frac{1}{8(26)} - \frac{1}{8} + \frac{25}{8}\right)d\theta =$$

$$\int_0^{2\pi}\left(\frac{625}{208}\right)d\theta = 2\pi\left(\frac{625}{208}\right) = \boxed{\frac{625\pi}{104}}$$