10) Define $f(x,t) = \dfrac{e^{\left(\frac{kx^2}{t}\right)}}{\sqrt{t}}$ where k is a constant. There is one value of k for which this function is a solution of the Heat Equation, $f_{xx} = f_t$. Find the value of k and verify that the resulting function does solve the equation.

$$f_x = \frac{2xk\, e^{\left(\frac{kx^2}{t}\right)}}{t\sqrt{t}} = \frac{2xk}{t\sqrt{t}} \cdot e^{\left(\frac{kx^2}{t}\right)}$$

Product Rule

$$\frac{2xk}{t\sqrt{t}} \rightarrow \frac{2k}{t\sqrt{t}}$$

$$e^{\left(\frac{kx^2}{t}\right)} \rightarrow \frac{2xk}{t} e^{\left(\frac{kx^2}{t}\right)}$$

$$f_{xx} = \left(\frac{2k}{t\sqrt{t}}\right)\left(e^{\left(\frac{kx^2}{t}\right)}\right) + \left(\frac{2xk}{t\sqrt{t}}\right)\left(\frac{2xk}{t} e^{\left(\frac{kx^2}{t}\right)}\right)$$

$$f_{xx} = \frac{2k e^{\left(\frac{kx^2}{t}\right)}}{(t)^{3/2}} + \frac{4x^2k^2 e^{\left(\frac{kx^2}{t}\right)}}{(t)^{5/2}}$$

Product Rule

$$t^{-1/2} \rightarrow -\tfrac{1}{2} t^{-3/2}$$

$$e^{\left(\frac{kx^2}{t}\right)} \rightarrow e^{\left(\frac{kx^2}{t}\right)} \cdot \frac{-kx^2}{t^2}$$

$$f(x,y) = \left(\frac{1}{\sqrt{t}}\right)\left(e^{\left(\frac{kx^2}{t}\right)}\right)$$

$$f_t = \left(\frac{-1}{2(t)^{3/2}}\right)\left(e^{\frac{kx^2}{t}}\right) + \left(\frac{1}{\sqrt{t}}\right)\left(\frac{-kx^2}{t^2}\right)\left(e^{\left(\frac{kx^2}{t}\right)}\right)$$

$$f_t = \left(\frac{-e^{\left(\frac{kx^2}{t}\right)}}{2(t)^{3/2}}\right) - \left(\frac{kx^2 e^{\left(\frac{kx^2}{t}\right)}}{(t)^{5/2}}\right)$$

$$f_t = \left[\left(\frac{-1}{2(t)^{3/2}}\right) - \left(\frac{kx^2}{(t)^{5/2}}\right)\right]\left(e^{\left(\frac{kx^2}{t}\right)}\right)$$

$$f_{xx} = \left[\left(\frac{2k}{(t)^{3/2}}\right) + \left(\frac{4x^2k^2}{(t)^{5/2}}\right)\right]\left(e^{\left(\frac{kx^2}{t}\right)}\right)$$