Answers to

Quiz #9 for Math 250:1 & 5

4/25/2011

Name ______________________________ **Section** (please circle one) 1 5

Vectors $v_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -1 \end{bmatrix}$, and $v_3 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ span a 3-dimensional subspace S of $\mathbb{R}^4$.

1. (7) Find an orthogonal basis $\{w_1, w_2, w_3\}$ for S.

Answer The Gram-Schmidt process recommends what follows.

First $w_1 = v_1 = \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$. **Second** $w_2 = v_2 - \dfrac{v_2 \cdot w_1}{w_1 \cdot w_1} w_1$. Here $v_2 \cdot w_1 = 2 \cdot (-1) + 0 \cdot (-1) + (-1) \cdot 0 + (-1) \cdot 1 = -3$ and $w_1 \cdot w_1 = (-1) \cdot (-1) + (-1) \cdot (-1) + 0 \cdot 0 + 1 \cdot 1 = 3$ so $w_2 = v_2 - \left(-\frac{3}{3}\right) w_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix}$. **Third** $w_3 = v_3 - \dfrac{v_3 \cdot w_1}{w_1 \cdot w_1} w_1 - \dfrac{v_3 \cdot w_2}{w_2 \cdot w_2} w_2$.
Here $v_3 \cdot w_1 = (-1) \cdot (-1) + 1 \cdot (-1) + 1 \cdot 0 + 1 \cdot 1 = 1$, $w_1 \cdot w_1 = 3$ again, $v_3 \cdot w_2 = (-1) \cdot 1 + 1 \cdot (-1) + 1 \cdot (-1) + 1 \cdot 0 = -3$, and $w_2 \cdot w_2 = 1 \cdot 1 + (-1) \cdot (-1) + (-1) \cdot (-1) + 0 \cdot 0 = 3$.
so $w_3 = v_3 - \frac{1}{3} w_1 - \left(\frac{-3}{3}\right) w_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \\ -\frac{1}{3} \end{bmatrix} + \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix}$.

2. (3) The vector $w = \begin{bmatrix} 4 \\ -2 \\ -3 \\ -2 \end{bmatrix}$ is in S and $w = aw_1 + bw_2 + cw_3$. Find a, b, and c.

Answer $a = \dfrac{w \cdot w_1}{w_1 \cdot w_1} = \dfrac{(4 \cdot (-1) + (-2) \cdot (-1) + (-3) \cdot 0 + (-2) \cdot 1)}{3} = -\dfrac{4}{3}$.
$b = \dfrac{w \cdot w_2}{w_2 \cdot w_2} = \dfrac{(4 \cdot 1 + (-2) \cdot (-1) + (-3) \cdot (-1) + (-2) \cdot 0)}{3} = \dfrac{9}{3} = 3$.
$c = \dfrac{w \cdot w_3}{w_3 \cdot w_3} = \dfrac{\left(4 \cdot \left(\frac{1}{3}\right) + (-2) \cdot \left(\frac{1}{3}\right) + (-3) \cdot 0 + (-2) \cdot \left(\frac{2}{3}\right)\right)}{\left(\left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) + \left(\frac{1}{3}\right) \cdot \left(\frac{1}{3}\right) + 0 \cdot 0 + \left(\frac{2}{3}\right) \cdot \left(\frac{2}{3}\right)\right)} = -1$.

Comment We can check these answers but I would not expect students to do this in the context of a brief quiz. Here are the details: $-\frac{4}{3} w_1 + 3w_2 + (-1)w_3 = -\frac{4}{3} \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} + (-1) \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ 0 \\ \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ -3 \\ -2 \end{bmatrix} = w$ as desired.