Answers to

Quiz #8B for Math 250:1 & 5

4/6/2011

Name ______________________ **Section** (please circle one) 1 5

In these problems, A is the 4×4 matrix $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

1. (8) a) Find the characteristic polynomial of A and the eigenvalues of A.

Answer The determinant of $A-\lambda I_4$ is $(2-\lambda)^2(-1-\lambda)(-\lambda)$ (the matrix is upper-triangular so the determinant is the product of the diagonal elements). The eigenvalues are 0, -1, and 2.

b) Find a basis for each eigenspace of A.

Answer $\underline{\lambda = 0}$. Then $A-\lambda I_4$ is $\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. The third row implies that $x_3 = 0$ and then the first row implies that $x_1 = 0$. The fourth row has no information but the second row shows that $x_4 = -2x_2$. A basis for this eigenspace is $\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \\ -2 \end{bmatrix} \right\}$.

$\underline{\lambda = -1}$. Then $A-\lambda I_4$ is $\begin{bmatrix} 3 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. The last row shows that $x_4 = 0$ and then the second row shows that $x_2 = 0$. The third row has no information. The first row implies that $x_3 = -3x_1$. A basis for this eigenspace is $\left\{ \begin{bmatrix} 1 \\ 0 \\ -3 \\ 0 \end{bmatrix} \right\}$.

$\underline{\lambda = 2}$. Then $A-\lambda I_4$ is $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$. The last row shows that $x_4 = 0$, the third row shows that $x_3 = 0$, and the first and second rows repeat that information. Therefore both x_1 and x_2 are not controlled: they are *free*. A basis for this eigenspace is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}$.

2. (2) Can A be diagonalized? Briefly support your assertion with some reasoning.

Answer <u>Yes</u> A has *four* linearly independent eigenvectors in $\mathbb{R}^4$ and this is what is needed for diagonalization.