

Name _____ Section (please circle one) 1 5

1. (4) In this problem, A is an $n \times n$ matrix, and v is a vector in \mathbb{R}^n . Define v is an eigenvector of A with corresponding eigenvalue λ .

Answer A non-zero vector v is an eigenvector of A if $Av = \lambda v$ for some scalar λ . Here λ is the eigenvalue corresponding to v .

2. (6) In this problem A is a 4×4 matrix, and v is an eigenvector of A with corresponding eigenvalue 3. You do *not* need to justify your answers in this problem.

Comment The equation $Av = 3v$ is used in the explanations which follow. This equation “encodes” the information given. Also, since v is an eigenvector, it is not zero which implies that the vectors considered in the situations below are also not zero.

a) Is $-2v$ an eigenvector of A ? If it is, what is its corresponding eigenvalue?

Answer YES Just consider $A(-2v) = -2Av = (-2)3v = 3(-2v)$. Therefore $-2v$ is an eigenvector of A and its corresponding eigenvalue is $\lambda = 3$.

b) Is v an eigenvector of $5A$? If it is, what is its corresponding eigenvalue?

Answer YES Just consider $(5A)v = 5(Av) = 5(3v) = (15)v$. Therefore v is an eigenvector of $5A$ and its corresponding eigenvalue is $\lambda = 15$.

c) Is v an eigenvector of A^2 ? If it is, what is its corresponding eigenvalue?

Answer YES Just consider $A^2v = (AA)v = A(Av) = A(3v) = 3(Av) = 3(3v) = 9v$. Therefore v is an eigenvector of A^2 and its corresponding eigenvalue is $\lambda = 9$.