Quiz #6 for Math 250:1 & 5  

3/23/2011

Name ____________________________  Section (please circle one)  1  5

1. (4) In this problem, $S$ is a subspace of $\mathbb{R}^n$ which is **not** \{0\}.

   a) Define *basis* of $S$.

   **Answer** A basis of $S$ is a linearly independent set of vectors in $S$ which also spans $S$.

   b) Define *dimension* of $S$.

   **Answer** The dimension of $S$ is the number of vectors in a basis of $S$.

2. (3) Suppose $S$ is the subspace of $\mathbb{R}^4$ that is the span of \[
\begin{bmatrix}
1 \\
0 \\
1 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
1 \\
2 \\
0 \\
0 \\
\end{bmatrix}, \begin{bmatrix}
2 \\
0 \\
2 \\
0 \\
\end{bmatrix} \]. Find the dimension of $S$ and give a basis of $S$. You do **not** need to justify your answers.

   **Answer** If $v_1$, $v_2$, and $v_3$ are the vectors given, then $v_3 = 2v_1$ so we can discard $v_3$ without changing the span. Notice that $v_1$ and $v_2$ are linearly independent (consider the second and third coordinates, for example). Therefore $S$ has dimension 2 and a basis of $S$ is given by $v_1$ and $v_2$.

3. (3) Suppose $S$ is a subspace of $\mathbb{R}^{500}$ which contains a set of 4 linearly independent vectors and which is spanned by 6 of its vectors. What are the possible values of the dimension of $S$? You do **not** need to justify your answer.

   **Answer** Any linearly independent set can be increased to a basis, and any spanning set can be “shrunk” to form a basis. Therefore a basis of $S$ will have at least 4 vectors and at most 6 vectors. The dimension of $S$ must be 4, 5, or 6.