Some answers to **Quiz #5 for Math 250:1 & 5** 3/9/2011

Name ______________________________ **Section** (please circle one) 1 5

$$\begin{bmatrix} 3 & -2 & 6 & 4 & a \\ 5 & 3 & 2 & 1 & b \\ 1 & -7 & 10 & 7 & c \\ 12 & 11 & 0 & -1 & d \end{bmatrix} \text{ is (after row ops) } \begin{bmatrix} 1 & 0 & \frac{22}{19} & \frac{14}{19} & \frac{2}{19}b + \frac{3}{19}a \\ 0 & 1 & -\frac{24}{19} & -\frac{17}{19} & \frac{3}{19}b - \frac{5}{19}a \\ 0 & 0 & 0 & 0 & -2a + b + c \\ 0 & 0 & 0 & 0 & a - 3b + d \end{bmatrix}.$$

This problem discusses the following system of linear equations:

$$\begin{cases} 3x_1 - 2x_2 + 6x_3 + 4x_4 = a \\ 5x_1 + 3x_2 + 2x_3 + x_4 = b \\ x_1 - 7x_2 + 10x_3 + 7x_4 = c \\ 12x_1 + 11x_2 \qquad - x_4 = d \end{cases}$$

In these problems, you need not supply verification of your answers.

1. (2) Find specific numbers for which this system has <u>*no solution*</u>.

Answer The system will have no solution if either $c - 2a + b \neq 0$ or $d - a - 3b \neq 0$ because then one of the last two rows of the right-hand matrix above will represent the equation $0x_1 + 0x_2 + 0x_3 + 0x_4 =$ something not 0. An example with specific numbers is $a = 0$, $b = 0$, $c = 1$, and $d = 1$. Then both expressions are not equal to 0.

2. (3) Consider the collection of all possible $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ which are solutions for the associated <u>homogeneous</u> system. Write a <u>generating set</u> of <u>linearly independent</u> vectors for this span.

Answer If all of a, b, and c are 0, then the last two rows of the right-hand matrix above has no content ($0 = 0$ twice). The first two rows show that $x_1 = -\frac{22}{19}x_3 - \frac{14}{19}x_4$ and $x_3 = \frac{24}{19}x_3 + \frac{17}{19}x_4$. Then $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{22}{19}x_3 - \frac{14}{19}x_4 \\ \frac{24}{19}x_3 + \frac{17}{19}x_4 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -\frac{22}{19} \\ \frac{24}{19} \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -\frac{14}{19} \\ \frac{17}{19} \\ 0 \\ 1 \end{bmatrix}$.

Therefore a generating set of linearly independent vectors is $\left\{ \begin{bmatrix} -\frac{22}{19} \\ \frac{24}{19} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{14}{19} \\ \frac{17}{19} \\ 0 \\ 1 \end{bmatrix} \right\}$.

3. (2) Find specific numbers (*not all 0!*) for which this system has a solution, and display one specific solution.

Answer We need to know that $c - 2a + b = 0$ and $d - a - 3b = 0$ so that both of the two rows of the right-hand matrix represent the equation $0 = 0$. One simple example is $a = 0$, $b = 1$, $c = -1$, and $d = 3$. If we set the free variables x_3 and x_4 both to be 0, then the first two rows of the right-hand matrix imply that $x_1 = \frac{2}{19}$ and $x_2 = \frac{3}{19}$.

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4. (3) Consider the collection of all possible $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ for which this system has a solution. Write a generating set of linearly independent vectors for this span.

Answer Again, we need to know that $c - 2a + b = 0$ and $d - a - 3b = 0$ so that both of the two rows of the right-hand matrix represent the equation $0 = 0$. Therefore $c = 2a - b$ and $d = -a + 3b$ (the matrix corresponding to these "compatibility conditions" is in RREF, actually). Then $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a \\ b \\ 2a - b \\ -a + 3b \end{bmatrix} = a \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \\ -1 \\ 3 \end{bmatrix}$ and a generating set of linearly independent vectors is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 3 \end{bmatrix} \right\}$.

Comments Of course, I hope that after the lecture of March 9 you will recognize what is described in #2 and #4. The solution to the former problem, where we deal with all solutions to the associated homogeneous system, is of course a concise (linear independence!) description of **Null** A (here A is the coefficient matrix of the linear system) using a linearly independent generating set. The linear independence happens because the entries $c - 2a + b$ and $d - a - 3b$ represent a system of equations which `Maple` has kindly (automatically?) put in RREF. The latter problem, #4, asks for the column space or range of the coefficient matrix. One answer is to write the generating set as the collection of columns, $\left\{ \begin{bmatrix} 3 \\ 5 \\ 1 \\ 12 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 7 \\ 11 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 10 \\ 0 \end{bmatrix}, \begin{bmatrix} 12 \\ 11 \\ 0 \\ -1 \end{bmatrix} \right\}$. This doesn't answer #4 adequately since these vectors are not *linearly independent*. This description is redundant (wasteful!). The RREF displayed can be used to find several specific non-trivial linear combinations of the four vectors which are 0. However, our "procedure" gives a nice description of a generating set.