Quiz #4 for Math 250:1 & 5 2/23/2011

Name ___________________________  Section (please circle one)  1  5

1. (2) Suppose that \( A \) is an \( m \times n \) matrix. Define the rank of \( A \) and the nullity of \( A \).

   **Suggestion** Use complete English sentences. You might want your first sentence to begin
   with the phrase: “If \( R \) is the reduced row echelon form of \( A \), then the ...”.

   Suppose \( R \) is the reduced row echelon form of \( A \), then the rank of \( A \) is the number of
   pivots in \( R \) and the nullity of \( A \) is \( n - \) the rank of \( A \).

   **Simple examples are better than complicated examples!**

2. (2) Give an example of a matrix which has rank = 3 and nullity = 2. You need not verify your example!

   **Answer**
   \[
   \begin{bmatrix}
   1 & 0 & 0 & 0 & 0 \\
   0 & 1 & 0 & 0 & 0 \\
   0 & 0 & 1 & 0 & 0
   \end{bmatrix}
   \]

3. (2) Give two \( 2 \times 2 \) matrices \( A \) and \( B \) so that the rank of \( A \) is 1, the rank of \( B \) is 1, and
   the rank of \( A + B \) is 2. You need not verify your example!

   **Answer** \( A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \) so that \( A + B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \).

4. (2) Give two \( 2 \times 2 \) matrices \( A \) and \( B \) so that the rank of \( A \) is 1, the rank of \( B \) is 1, and
   the rank of \( A + B \) is 1. You need not verify your example!

   **Answer** \( A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) so that \( A + B = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \).

5. (2) Give two \( 2 \times 2 \) matrices \( A \) and \( B \) so that the rank of \( A \) is 1, the rank of \( B \) is 1, and
   the rank of \( A + B \) is 0. You need not verify your example!

   **Answer** \( A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \) and \( B = \begin{bmatrix} -1 & 0 \\ 0 & 0 \end{bmatrix} \) so that \( A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \).