#7 **Problems for 153** 10/21/2009

1. The following statements are facts:

$$(10{,}000{,}000{,}000)^{\left(\frac{1}{10{,}000{,}000{,}000}\right)} \approx 1.\ 00000\ 000\underline{23}\ \underline{02585}\ \underline{09}564 \textbf{ and } \ln 10 \approx \underline{2}.\underline{30258}\ \underline{509}29\,.$$

Explain the amazing coincidence of the digits.

Hint Approximate e^x when x is small.

2. Example 1 in section 3.11 of the text analyzes the following problem:

> A 16-ft ladder leans against a wall. The bottom of the ladder is 5 ft from the wall at time $t = 0$ and slides away from the wall at a rate of 3 ft/s. Find the velocity of the top of the ladder at time $t = 1$.

a) The textbook response to this question is $-\sqrt{3} \approx -1.732$ ft/s. The minus sign means the top of the ladder is sliding *down*. Check that the textbook's answer is correct.

b) The speed of sound at sea level using the standard atmosphere is about 340.29 meters per second. There are 3.280840 feet in one meter. Using the assumptions of this model, find the angle between the ladder and the ground at the time that the top of the ladder breaks the speed of sound.

c) The speed of light is about 299,792,458 meters per second. There are still 3.280840 feet in one meter. Using the assumptions of this model, find the angle between the ladder and the ground at the time that the top of the ladder moves at the speed of light.

3. The numbers R_1, R_2, R_3, and R satisfy this equation: $\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} = \dfrac{1}{R}$.
(Physics and engineering students may recognize this as a formula for the total resistance, R, of a circuit composed of three resistances R_1, R_2, and R_3 connected in parallel.)

a) If $R_1 = 1$ and $R_2 = 2$ and $R_3 = 3$, compute R exactly.

b) If R_1 and R_3 are constant, and R_2 is increased by .05, approximate the change in R.

c) If R_1 and R_2 are constant, and R_3 is increased by .05, approximate the change in R.

4. In this problem, $f(x) = \dfrac{1}{1+x} - \cos x$.

a) Graph $f(x)$ in the window $0 \le x \le 6$ and $-1 \le y \le 1.5$.

b) Write an equation showing how x_n, an approximation for a root of $f(x) = 0$, is changed to an improved approximation, x_{n+1}, using Newton's method. Your equation should use the specific function in this problem.

c) Suppose $x_0 = 2$. Compute the next two approximations x_1 and x_2. Explain what happens to the sequence of approximations $\{x_n\}$ as n gets large. You should use both numerical and graphical evidence to support your assertion.

d) Suppose $x_0 = 4$. Compute the next two approximations x_1 and x_2. Explain what happens to the sequence of approximations $\{x_n\}$ as n gets large. You should use both numerical and graphical evidence to support your assertion.