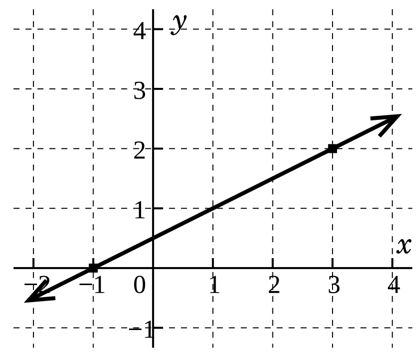
Answers to the 153 Diagnostic Exam

1. Write $\left(3x^2 - \dfrac{4}{x}\right)(2x^3 + 4x)$ as a polynomial in standard form ("expanded").

$\left(3x^2 - \dfrac{4}{x}\right)(2x^3 + 4x) = 3x^2(2x^3 + 4x) - \left(\dfrac{4}{x}\right)(2x^3 + 4x) = 6x^5 + 12x^3 - 8\left(\dfrac{x^3}{x}\right) - 16\left(\dfrac{x}{x}\right) =$
$6x^5 + 12x^3 - 8x^2 - 16$

The polynomial is $\underline{6x^5 + 12x^3 - 8x^2 - 16}$

2. Find an equation for the line through the points $(-1, 0)$ and $(3, 2)$. **Sketch the line** on the axes to the right.
If $y = mx + b$ then $m = \frac{2-0}{3-(-1)} = \frac{2}{4} = \frac{1}{2}$. Insert $(-1, 0)$ in $y = \frac{1}{2}x + b$ to get $0 = -\frac{1}{2} + b$ so $b = \frac{1}{2}$. Check with $(3, 2)$: $2 = \frac{1}{2}(3) + b$ so $b = \frac{1}{2}$ again.



An equation for the line is $\underline{\quad y = \frac{1}{2}x + \frac{1}{2} \quad}$

3. Suppose $f(x) = x^2 + 3$.
a) Then $f(2) = \underline{\quad 7 \quad}$ and $f\left(-\frac{1}{3}\right) = \underline{\quad \frac{28}{9} \quad}$.

$f(2) = 2^2 + 3 = 7$ and $f(\frac{1}{3}) = \frac{1}{9} + 3 = \frac{28}{9}$ but $3\frac{1}{9}$ is also a correct answer.

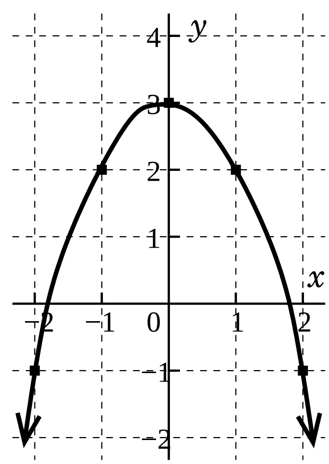
b) Find a formula for $f(f(x))$ (with *no mention of* f!) which is correct for any value of x.
$f(f(x)) = f(x^2 + 3) = \left(x^2 + 3\right)^2 + 3$, a fine answer, which can be further "simplified" to $x^4 + 6x^2 + 9 + 3 = x^4 + 6x^2 + 12$.

A formula for $f(f(x))$ **is** $\underline{\quad x^4 + 6x^2 + 12 \quad}$

4. If $g(x) = -x^2 + 3$, **sketch a graph** of $y = g(x)$ on the axes to the right.

$g(0) = -0^2 + 3 = 3$ so $(0, 3)$ is on the graph.
$g(1) = -1^2 + 3 = 2$ so $(1, 2)$ is on the graph.
$g(2) = -2^2 + 3 = -1$ so $(2, -1)$ is on the graph.
$g(-1) = -1^2 + 3 = 2$ so $(-1, 2)$ is on the graph.
$g(0) = -2^2 + 3 = -1$ so $(-2, -1)$ is on the graph.

The graph is symmetric with respect to the y-axis, "opens" down, and extends beyond the interval $-2 \le x \le 2$.



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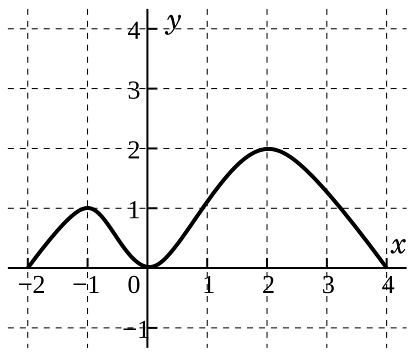
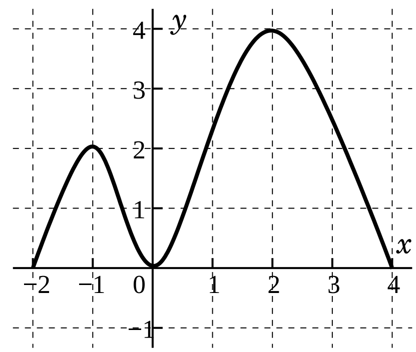
5. Below to the left is a graph of $y = h(x)$. **Sketch a graph** of $y = 2h(x)$ on the other axes below.



Graph of $y = h(x)$



Graph of $y = 2h(x)$

The "2" on the *outside* of $h(x)$ doubles the height, y, of the graph. The x-intercepts are still at -2, 0, and 4. The intervals where the function increases and decreases are not changed, just stretched.

6. Inside the rectangle $ABCD$ and sharing a right angle with it is another rectangle as shown to the right. P and Q are the lengths of two consecutive sides of the inner rectangle.

Write an expression for the area of rectangle $ABCD$ **involving** P **and** Q **and other needed numbers and operations.** Use any of the information displayed.

You do *not* need to "simplify" your expression!

The length of AB is $P + 2$ and the length of BC is $Q + 3$. The area is the product of these lengths: $(P + 2)(Q + 3)$. This is a fine answer. Actually, to me the answer $PQ + 2Q + 3P + 6$ is *too* complicated, and makes the solution more difficult to understand!

The area of rectangle $ABCD$ **is** $(P + 2)(Q + 3)$