1. Sketch the three-sided region in the first quadrant bounded by the $y$-axis and the two curves $y = \tan x$ and $y = \sec x$. Compute the area of this region.

2. A computer program reports the following:

$$\int_{0}^{1} \frac{x}{x+1} \, dx = 1 - \ln 2; \quad \int_{0}^{\infty} \frac{t}{(2t+1)(t+1)^2} \, dt = 1 - \ln 2.$$ 

Verify that the two integrals are equal. Notice that you are not asked to evaluate these definite integrals, only to explain why the values are equal.

**Hint** Find the antiderivatives and compute both integrals: a very direct method.

**(Hint)** Change one integral into the other: $x$ goes from 0 to 1 and $t$, from 0 to $\infty$ — everything involved is a rational function, so make the change from $x$ to $t$ with a simple rational function. After you find a suitable change of variables, how does $dx$ change to $dt$?

3.* Find the limits for the following indeterminate forms of the type “$\infty - \infty$”.

a) \( \lim_{x \to 0} \frac{1}{\sin x} - \frac{1}{x} \).

b) \( \lim_{x \to 0} \frac{1}{x^2} - \frac{1}{x} \).

c) \( \lim_{x \to 0} \frac{1+x}{x} - \frac{1-x}{x} \).

4. Suppose that $a$ is a positive constant and that $R$ is the region bounded above by $y = \frac{1}{x^a}$, below by $y = 0$, and on the left by the line $x = 1$.

a) Sketch the curves $y = \frac{1}{x^a}$ for $a = .5, 1$ and 2. Which of these is closest to the $x$-axis?

b) For which positive numbers $a$ do you get a convergent integral when you attempt to calculate the area of $R$?

c) Same as b), but for the volume of the solid obtained by rotating $R$ around the $x$-axis.

d) Same as c), but for the volume of the solid obtained by rotating $R$ around the $y$-axis.

5. The curve $y = e^{-x}$, $x \geq 0$, is revolved about the $x$-axis. Does the resulting surface have finite or infinite area? (Remember that you can sometimes decide whether an improper integral converges without calculating it exactly.)

* For those to whom L'Hôpital's Rule is new.