1. Graph the functions \( f(x) = \frac{4x}{x^2+1} \) and \( g(x) = 2 \sin(2 \arctan x) \) in the same viewing window over the interval \(-5 \leq x \leq 5\). Explain what you see.

2. Suppose that \( f(x) = e^{-Ax} \), where \( A \) is a positive real number.
   a) Show that the integral \( \int_1^2 f(x) \, dx \to 0 \) as \( A \to \infty \). (You may wish to draw a picture, but other verification is also necessary.)
   b) Show that the integral \( \int_1^2 xf(x) \, dx \to 0 \) as \( A \to \infty \). (You may wish to draw a picture, but other verification is also necessary.)
   c) Show that the integral \( \int_1^2 x^2f(x) \, dx \to 0 \) as \( A \to \infty \). (You may wish to draw a picture, but other verification is also necessary.)

**Note** It isn’t always necessary or even possible to compute every integral exactly. But this integral can be estimated to get enough information.

3. a) Compute \( \int_0^{2\pi} (\cos(mx))(\cos(nx)) \, dx \) if \( m \) and \( n \) are integers.
   (Be careful: there are two different results, one when \( m = n \) and one when \( m \neq n \).)
   b) If \( f(x) = A \cos(x) + B \cos(2x) + C \cos(3x) \), \( \int_0^{2\pi} f(x) \cos(x) \, dx = 5 \), \( \int_0^{2\pi} f(x) \cos(2x) \, dx = 6 \), and \( \int_0^{2\pi} f(x) \cos(3x) \, dx = 7 \), then find \( A \) and \( B \) and \( C \).

**Note** The ideas of this computation are used often with Fourier series, a standard method of analyzing periodic phenomena. A graph of \( f \) is shown to the right. Your ear (and some mechanical and electrical devices) can find 5 and 6 and 7 in this graph!

4. a) Suppose \( A \) is a positive real number and \( m_A \) is the average value of \( (\sin(Ax))^3 \) on the interval \([0, 2]\). Compute \( m_A \).

**Note** The answer will have several terms and will not be simple.

b) What is \( \lim_{A \to \infty} m_A \) ?

**Note** This answer should be simple. Explain briefly why it is correct. You may refer to graphs of functions if that is helpful.

5. An oil tank has the shape of a cylinder whose diameter is 4 feet. It is mounted so that the axis of the cylinder is horizontal (the circular cross-sections of the cylinder are vertical). If the depth of the water is 3 feet, what percentage of the total capacity of the tank is filled?

After drawing a picture and setting up this problem, solve it three ways:

a) Use elementary geometry (compare areas of circular sectors).

b) Express the answer in terms of a definite integral, then obtain an approximate numerical value for the integral using the \texttt{fnInt}\( f \) function on your calculator.

c) Evaluate the integral in b) exactly in terms of elementary functions using a trig substitution, then obtain approximate numerical values for these functions using your calculator.