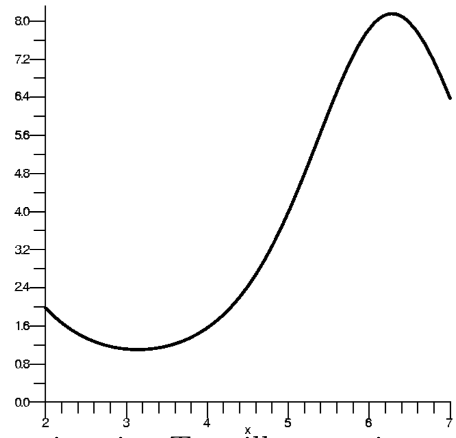
1. Suppose f is defined by $f(x) = 3e^{\cos x}$. `Maple` produced graphs of f and its first four derivatives on the interval $[2, 7]$ (be careful when examining the derivative graphs – look carefully at the vertical scales!). The graph of f is to the right, and the graphs of the first four derivatives of f are <u>on the back of this page</u>. You should assume that the graphs given are correct.
Suppose I is the value of $\int_2^7 f(x)\,dx$.



a) Use the graph of f alone to estimate I.

b) Use the information in the graphs to tell how many subdivisions N are needed so that the Trapezoid Rule approximation T_N will approximate I with error $< 10^{-5}$.

c) Use the information in the graphs to tell how many subdivisions N are needed so that the Simpson's Rule approximation S_N will approximate I with error $< 10^{-5}$.

2. Consider the function $f(x) = e^x \sin(Nx)$ <u>on the interval</u> $[0, 1]$ where N is a positive integer.

a) Make three separate sketches of the graph of this function when $N = 5$, $N = 10$, and $N = 100$. (Use a calculator and try to make "good" sketches.)

b) Compute $\int_0^1 f(x)\,dx$ in general, for any positive integer N. Evaluate the result when $N = 5$, $N = 10$, and $N = 100$.

c) What happens to the graph and to the value of the integral as $N \to \infty$? Explain how the graphs confirm the limiting behavior of the integral's value.

3. The only information known about a function T and its derivatives is contained in this table:

x	$T(x)$	$T'(x)$	$T''(x)$
1	2	-2	2
2	3	6	5
3	7	4	-4
4	2	5	7

a) Compute $\int_2^3 T'(x)\,dx$.

b) Compute $\int_2^3 T''(x)\,dx$.

c) Compute $\int_2^3 x\,dx$.

d) Compute $\int_2^3 xT''(x)\,dx$. Don't look at b) and c)! Integrate by parts.

e) Compute $\int_2^3 x^2T'''(x)\,dx$. And again and again.

4. a) For x near 0, $\sin x$ is well-approximated by its tangent line at $x = 0$. What is this tangent line?

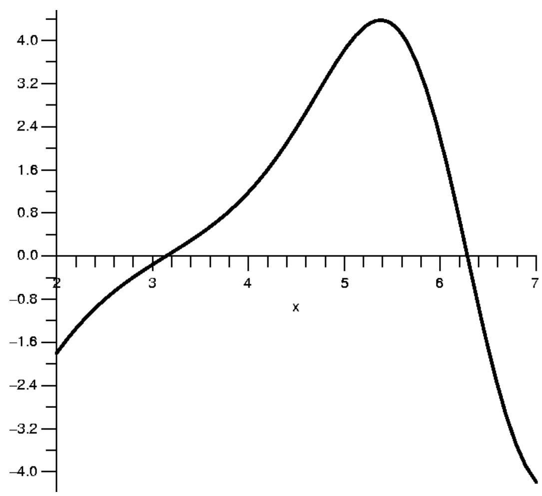
This problem continues on the other side of the paper.

b) Approximation over an interval is preferred over approximation near a point for many purposes. One criterion for assessing the accuracy of such an approximation is *mean-square error*. The mean-square error between a straight line $y = Ax$ going through the origin and the function $\sin x$ over the interval $[0, 1]$ is given by the definite integral $\int_0^1 (\sin x - Ax)^2 \, dx$. Find the A which <u>minimizes</u> this integral.

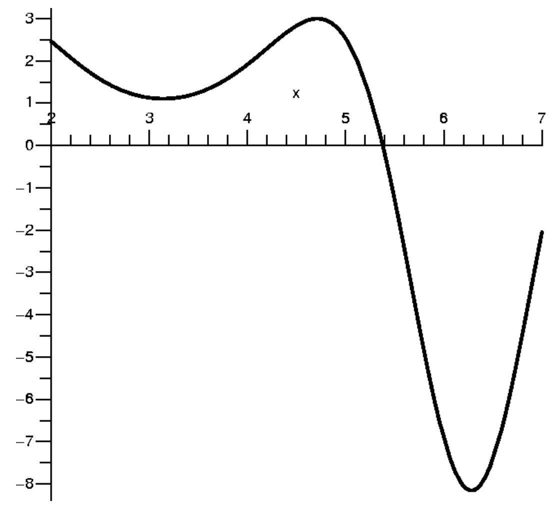
Hint Expand the integrand, compute the integral, and find the A minimizing the result.

c) Sketch $\sin x$ and the straight lines found in a) and b) on the unit interval $[0, 1]$.

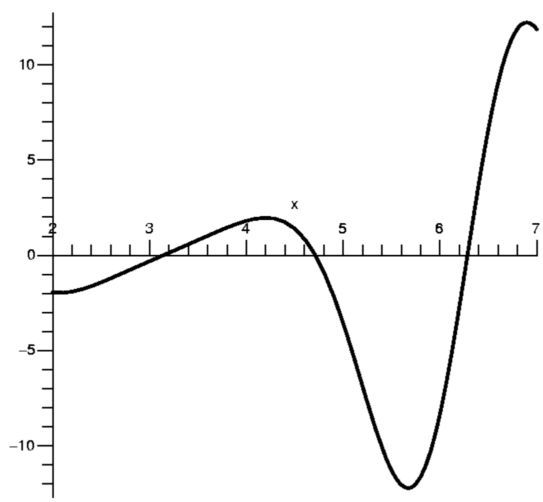
Graphs for problem 1



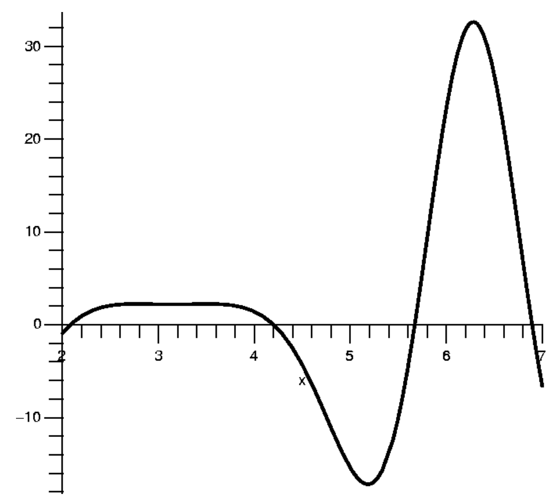
Graph of f'



Graph of f''



Graph of $f^{(3)}$



Graph of $f^{(4)}$