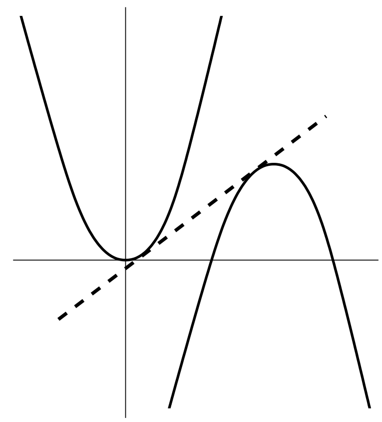
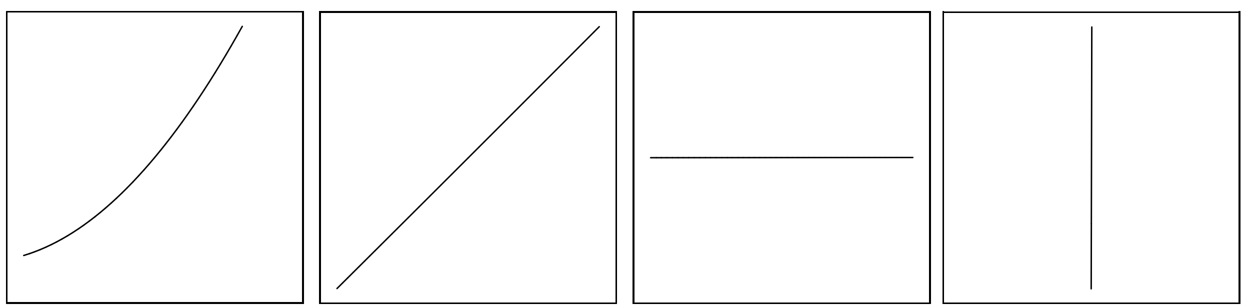
1. Consider the parabola $y = x^2$. Flip it and move it right, to create a parabola opening "down" and intersecting the x-axis at $x = 1$ and $x = 2$. An equation for such a parabola is $y = -(x-1)(x-2)$. To the right is a rough sketch of the two parabolas and a straight line tangent to both of the parabolas. Find an equation for that line. Include *any* other information (labeled pictures) that you think is useful.



Comment Sometimes my problems, perhaps imitating "real life", may be ... deceptively stated. Or even confusing. So part of the problem is making sense out of the problem. Also this becomes a good excuse for sloppiness.

2. Here are four graphs of $y = x^2$ all "drawn" by a computer. All of the windows are centered on the point $(2, 4)$. Find windows which could have produced the graphs shown, and explain your answers. Also, give one example of an approximately "straight line" graph which could *not* be produced by choosing a window centered around $(2, 4)$ and looking at $y = x^2$.



Graph #1 **Graph #2** **Graph #3** **Graph #4**

3. Find the area enclosed by the curves $y = c - x^2$ and $y = x^2 - c$ as a function of c. Find the value of c for which this area is equal to 1.

Note This is problem 50 in section 6.1 of the textbook.

4. Suppose f is a function defined by $f(x) = \displaystyle\int_0^x \frac{1}{\sqrt{1+t^3}}\,dt$ for $x > -1$ and g is the function inverse to f. Then g satisfies a differential equation of the following form: $g''(x) = kg(x)^2$ where k is a constant. What is k?

Hint What is f'? What is $f(g(x))$? Differentiate and then again.

Your assignment

Hand in on Wednesday, September 9, writeups for *two* of these problems.

Include in your solutions any information, including any pictures (properly labeled!) and computations that you think is useful. Your written solutions should be of high-quality, with the explanation of your solutions to problems given in complete English sentences. You will be graded on presentation as well as on mathematical content. Neatness counts (use staples!). While I encourage you to discuss the problem with other students and with me (either via e-mail or in person), the written work you hand in must be your own. The "local matter" in your textbook contains a discussion of writeups and a sample writeup. Details of routine computations generally don't need to be shown. Solutions don't need to be long (think of them as brief technical reports) but they must be understandable and correct. Solutions need *not* be typed, but multiple pages should be numbered. If you hand in more than one problem, please begin each problem on a separate page. What follows is another sample problem and its solution, with a few comments.

An example of a workshop problem solution

Here is a problem that was a candidate for your first assignment, mostly because it used the Mean Value Theorem.

> **Problem Statement** Suppose that $f'(x) = \frac{2}{1+x^4} - \frac{3}{4+x^4}$. Is $f(0) < f(1)$?
>
> **Note** It is not likely at this time that you can write a formula for a function with this derivative (and, by the way, such a formula wouldn't really help very much!). So you will have to make some indirect argument, just using the information you have about the derivative.
>
> **Solution** The answer is "Yes, $f(0) < f(1)$." Here is an explanation. One consequence of the Mean Value Theorem is the following result:
>
> > Suppose the derivative of a differentiable function is positive on an interval. Then the function is strictly increasing on that interval.
>
> We therefore will know that $f(0) < f(1)$ if we can verify that $\frac{2}{1+x^4} - \frac{3}{4+x^4} > 0$ for x in $[0,1]$. This inequality is equivalent to $\frac{2}{1+x^4} > \frac{3}{4+x^4}$. Since for x in $[0,1]$, both $1+x^4$ and $4+x^4$ are positive, this inequality is equivalent to $2(4+x^4) > 3(1+x^4)$ which simplifies to $5 > x^4$. This last inequality is certainly correct if x is in $[0,1]$.

$\longrightarrow$ Neatness, correctness, and ease of reading are important. $\longleftarrow$

By the way, a silicon friend found this function whose derivative is given by the formula in the problem statement:

$$\frac{\sqrt{2}}{4}\ln\left(\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right)+\frac{\sqrt{2}}{2}\arctan(\sqrt{2}x+1)+\frac{\sqrt{2}}{2}\arctan(\sqrt{2}x-1)$$
$$+\frac{3}{16}\ln(x^2-2x+2)-\frac{3}{8}\arctan(x-1)-\frac{3}{16}\ln(x^2+2x+2)-\frac{3}{8}\arctan(x+1)$$

Does knowing this formula help or is studying the derivative easier?