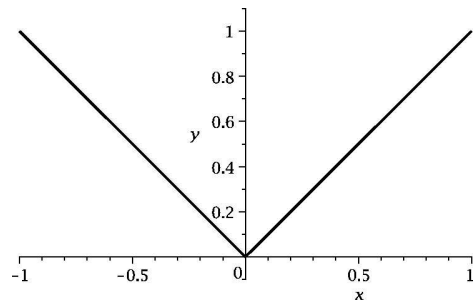
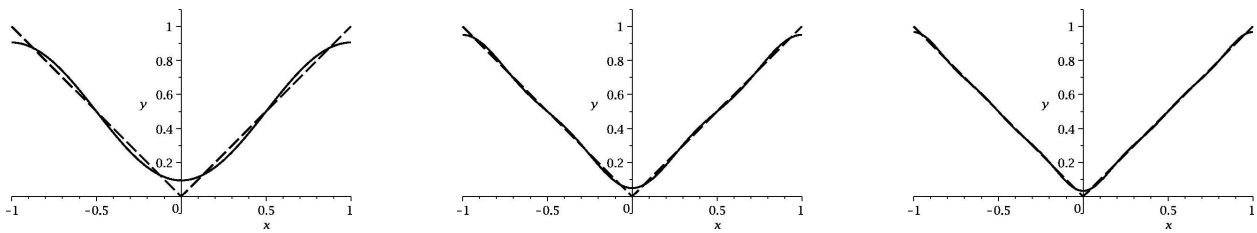
# Some pictures for 152H

10/21/2009

**Calculus** has been very successful in modeling phenomena and analyzing the resulting functions. But many *natural* models, however, seem to result in functions which are <u>not</u> differentiable, and therefore seem to lie outside the consideration of calculus. For example, $|x|$, the absolute value function on the interval $[-1, 1]$, is one such function. Maybe ... instead of studying $|x|$ directly, we could look at some *approximations*.



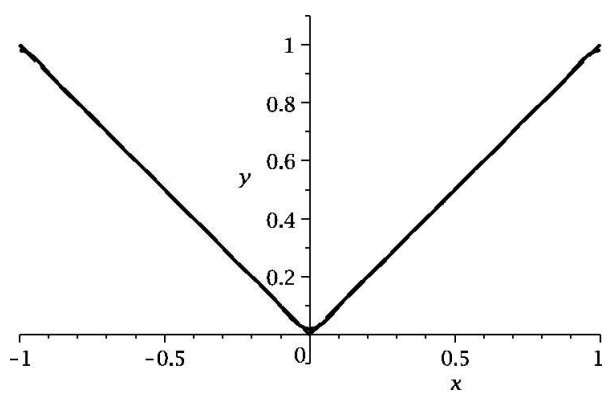

Below are three graphs. In each one, $|x|$ is shown as a two dashed line segments on $[-1, 1]$, and another function is sketched (solid curve) along with it.



The first function is $\underline{.5 - .40528\cos(\pi x)}$. The second function is $\underline{.5 - .40528\cos(\pi x) - .04503\cos(3\pi x)}$, and the last is $\underline{.5 - .40528\cos(\pi x) - .04503\cos(3\pi x) - .01621\cos(5\pi x)}$.

**→ What's going on, and where do these crazy numbers come from? ←**

The numbers come from ideas similar to what is in problem 5 of workshop 4. If one just ignores all sensibility and computes, then the coefficient multiplying $\cos(nx)$ in the functions above turns out to be $\displaystyle\int_{-1}^{1} |x|\cos(\pi n x)\,dx$. For this function, the coefficients for even positive $n$ are all 0 (*not* obvious!). The coefficient for $n = 0$ needs to be modified (divided by 2). But the resulting function is perfectly acceptable for calculus purposes. To the right is a graph of $|x|$ and the sum up to $\cos(9\pi x)$. They really do look very much the same.




This is really a graph of two functions.

Understanding this systematically needs some idea of how to approximate functions and how to take limits with functions. This is all covered in later studies of mathematics and its applications. We also will need to learn how to take sums and make the sums bigger: let the number of terms $\to \infty$. We will study this for the last third of Math 152.