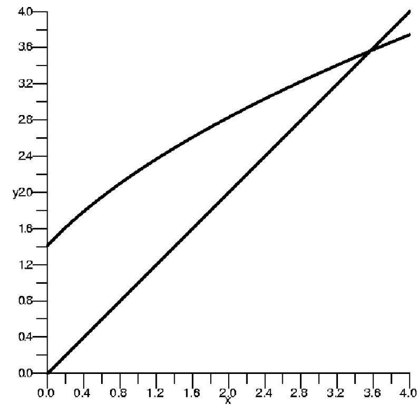
#9 **Problems for 152:01–03 and 152:06–09** 4/2/2009

This problem set has *four* problems.

1. Suppose $f(x) = \sqrt{2+3x}$, and suppose that the sequence $\{a_n\}$ has the following recursive definition:

$a_1 = 1$; $a_{n+1} = f(a_n)$ for $n > 1$.



a) Compute decimal approximations for the first 5 terms, a_1, a_2, a_3, a_4, and a_5, of the sequence.

b) The graph to the right shows parts of the line $y = x$ and the curve $y = \sqrt{2+3x}$. Locate on this graph or on a copy to be handed in the following points: (a_1, a_2), (a_2, a_2), (a_2, a_3), (a_3, a_3), (a_3, a_4), (a_4, a_4), (a_4, a_5), and (a_5, a_5). Also show a_1, a_2, a_3, a_4, and a_5 on the x-axis. (You must draw **13 points**.)

c) Write a statement of a result in section 10.1 which shows that this sequence converges. You must find a specific **THEOREM** in the section which will guarantee convergence.

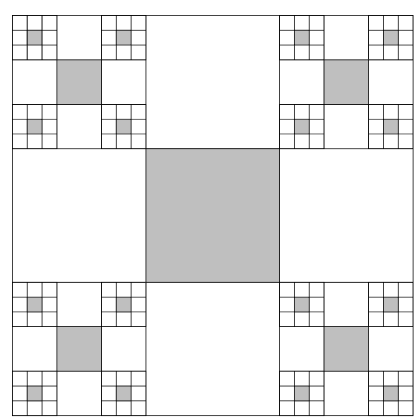
d) Compute the limit of $\{a_n\}$.

2. a) Two students are sharing a loaf of bread. Student Alpha eats half of the loaf, then student Beta eats half of what remains, then Alpha eats half of what remains, and so on. How much of the loaf will each student eat?

b) Two students are sharing a loaf of bread. Student Alpha, now hungrier and more ferocious, eats two-thirds of the loaf, then student Beta eats eats half of what remains, then Alpha eats two-thirds of what remains, then Beta eats half of what remains, and so on. How much of the loaf will each student eat?

c) Now start with three students: Alpha, Beta, and Gamma. They decide to share a loaf of bread. Alpha eats half of the loaf, passes what remains to Beta who eats half, and then on to Gamma who eats half, and then back to Alpha who eats half, and so on. How much of the loaf will each student eat?

3. A 1×1 square is "dissected" by three equally spaced horizontal lines and by three equally spaced vertical lines. The central square is shaded. Then the bordering Northeast, Northwest, Southeast, and Southwest squares are similarly dissected, with the central square shaded. Each of *those* dissected squares has a similar process done to their borders, etc. The diagram to the right shows this only for the first three steps but it is supposed to continue indefinitely.



a) How many new shaded squares are introduced at the n^{th} step? (There is one shaded square at the first step.) What is the side length of the squares which are introduced at the n^{th} step?

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b) What is the sum, as n goes from 1 to ∞, of the shaded area (all the shaded squares)? What is the sum, as n goes from 1 to ∞, of the perimeters of all the shaded squares?

4. A computer reports the following information:

$$\sum_{j=1}^{10} \frac{1}{j^3+2j^2+j} \approx 0.35105\,; \quad \sum_{j=1}^{100} \frac{1}{j^3+2j^2+j} \approx 0.35501\,; \quad \sum_{j=1}^{1000} \frac{1}{j^3+2j^2+j} \approx 0.35506\,.$$

This suggests that $\sum_{j=1}^{\infty} \frac{1}{j^3+2j^2+j}$ converges and that its sum is 0.355 (to 3 decimal places). Explain the details in the following outline of a verification of this statement.

a) The series has all positive terms. Therefore if the infinite tail $\sum_{j=101}^{\infty} \frac{1}{j^3+2j^2+j}$ converges and has sum less than .001, the omitted terms after the first 100 of the whole series won't matter to 3 decimal places.

b) Overestimate the infinite tail $\sum_{j=101}^{\infty} \frac{1}{j^3+2j^2+j}$ by the infinite tail of a simpler series. Then compare the infinite tail of this simpler series to a simple improper integral. Use a diagram to help explain the comparison. Compute the improper integral.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage <u>for this semester</u> to learn which problem to hand in.