1. The horizontal and vertical axes on this graph have different scales. \( x \) goes from \(-10\) to \(10\) and \( y \) goes from \(-1\) to \(3.5\). The graph is a direction field for the differential equation 
\[
y' = \frac{1}{10} \left(1 - \frac{1}{10} y x^2\right).
\]
a) Sketch the solution curve passing through \((0,1)\) on the graph.

b) How many critical points does this solution curve seem to have? What types of critical points do they seem to be? If \((x_0, y_0)\) is a critical point, find an exact algebraic relationship between \(x_0\) and \(y_0\).

Comment The equation can’t be solved in terms of standard functions. Use information from the graph and the differential equation.

2. Let \(N(t)\) be the fraction of the population who have heard a given piece of news \(t\) hours after its initial release. According to one model, the rate \(N'(t)\) at which the news spreads is equal to \(k\) times the fraction of the population that has not yet heard the news, for some constant \(k\).

a) Determine the differential equation satisfied by \(N(t)\).

b) Find the solution of this differential equation with the initial condition \(N(0) = 0\) in terms of \(k\).

c) Suppose that half of the population is aware of an earthquake 8 hours after it occurs. Use the model to calculate \(k\) and estimate the percentage that will know about the earthquake 12 hours after it occurs. (This is problem 22 in section 9.2 of the textbook.)

3. Air Resistance A projectile of mass \(m = 1\) travels straight up from ground level with initial velocity \(v_0\). Suppose that the velocity \(v\) satisfies \(v' = -g - kv\).

a) Find a formula for \(v(t)\).

b) Show that the projectile’s height \(h(t)\) is given by 
\[
h(t) = C(1 - e^{-kt}) - \frac{g}{k} t
\]
where 
\[
C = k^{-2}(g + kv_0).
\]

c) Show that the projectile reaches its maximum height at time \(t_{\text{max}} = k^{-1} \ln(1 + kv_0/g)\).

d) In the absence of air resistance, the maximum height is reached at time \(t = v_0/g\). In view of this, explain why we should expect that 
\[
\lim_{k \to 0} \frac{\ln(1 + \frac{kv_0}{g})}{k} = \frac{v_0}{g}.
\]

e) Verify the previous equation. (This is problem 26 in section 9.2 of the textbook. Look there for a \textit{Hint} about how to verify the equation.)