1. A computer program reports the following:

\[
\int_0^1 \frac{x}{x + 1} \, dx = 1 - \ln 2; \quad \int_0^\infty \frac{t}{(2t + 1)(t + 1)^2} \, dt = 1 - \ln 2.
\]

Verify that the two integrals are equal. Notice that you are not asked to evaluate these definite integrals, only to explain why the values are equal.

**Hint** Find the antiderivatives and compute both integrals: a very direct method.

**(Hint)** Change one integral into the other: \( x \) goes from 0 to 1 and \( t \), from 0 to \( \infty \) – everything involved is a rational function, so make the change from \( x \) to \( t \) with a simple rational function. After you find a suitable change of variables, how does \( dx \) change to \( dt \)?

2. Sketch carefully the graphs of \( f(x) = (1 + e^{-x})^2 \) and \( g(x) = (1 + e^{-2x})^2 \) for \( x > 0 \), and compute how much area there is between them in the first quadrant.

3. When a capacitor of capacitance \( C \) is charged by a source of voltage \( V \), the power expended at time \( t \) is \( P(t) = \frac{V^2}{R} \left( e^{-t/RC} - e^{-2t/RC} \right) \), where \( R \) is the resistance in the circuit. The total energy stored in the capacitor is \( W = \int_0^\infty P(t) \, dt \).

Show that \( W = \frac{1}{2} CV^2 \). (This is problem 81 in section 7.7 of the textbook.)

4. The curve \( y = e^{-x}, \ x \geq 0 \), is revolved about the \( x \)-axis. Does the resulting surface have finite or infinite area? (You can sometimes decide whether an improper integral converges without calculating it exactly.)

5. Sketch the three-sided region in the first quadrant bounded by the \( y \)-axis and the two curves \( y = \tan x \) and \( y = \sec x \). Compute the area of this region. Also compute the volume of the solid obtained when this region is revolved around the \( x \)-axis.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield’s Math 152 webpage for this semester to learn which problem to hand in.