1. A computer program reports the following:

$$\int_0^1 \frac{x}{x+1}\,dx = 1 - \ln 2\,; \qquad \int_0^\infty \frac{t}{(2t+1)(t+1)^2}\,dt = 1 - \ln 2\,.$$

Verify that the two integrals are equal. Notice that you are *not* asked to evaluate these definite integrals, only to explain why the values are equal.

Hint Find the antiderivatives and compute both integrals: a very direct method.

(Hint)^2 Change one integral into the other: x goes from 0 to 1 and t, from 0 to ∞ – everything involved is a rational function, so make the change from x to t with a simple rational function. After you find a suitable change of variables, how does dx change to dt?

2. Sketch carefully the graphs of $f(x) = (1+e^{-x})^2$ and $g(x) = (1+e^{-2x})^2$ for $x > 0$, and compute how much area there is between them in the first quadrant.

3. When a capacitor of capacitance C is charged by a source of voltage V, the power expended at time t is $P(t) = \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$, where R is the resistance in the circuit. The total energy stored in the capacitor is $W = \int_0^\infty P(t)\,dt$.
Show that $W = \frac{1}{2}CV^2$. (This is problem 81 in section 7.7 of the textbook.)

4. The curve $y = e^{-x}$, $x \geq 0$, is revolved about the x-axis. Does the resulting surface have finite or infinite area? (You can sometimes decide whether an improper integral converges without calculating it exactly.)

5. Sketch the three-sided region in the first quadrant bounded by the y-axis and the two curves $y = \tan x$ and $y = \sec x$. Compute the area of this region. Also compute the volume of the solid obtained when this region is revolved around the $x-$axis.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage for this semester to learn which problem to hand in.