1. Under the hypotheses of the integral test, if $a_n = f(n)$ then for any positive integer N, $\sum_{N+1}^{\infty} a_n \le \int_N^{\infty} f(x)\,dx$.

a) How large does N have to be to ensure that

(1) $\sum_{n=1}^{N} \frac{1}{n^5}$ is within 10^{-6} of $\sum_{n=1}^{\infty} \frac{1}{n^5}$?

(2) $\sum_{n=1}^{N} ne^{-n^2}$ is within 10^{-6} of $\sum_{n=1}^{\infty} ne^{-n^2}$?

b) Get a decimal approximation for the sum of one of the series with error less than 10^{-6}.

2. a) Verify that the infinite series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln(n))^2}$ converges. A computer gives the approximate value .84776 to 5 digit accuracy for the sum of this series. Find a specific partial sum which is guaranteed to give this number to 5 digit accuracy. Give evidence supporting your assertion.

b) Verify that the infinite series $\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln(n)}}$ diverges. A computer gives the approximate value of 4.74561 for the 10,000$^{\text{th}}$ partial sum. Are the partial sums of this series unbounded? If yes, find a specific partial sum which is guaranteed to be greater than 100. Give evidence supporting your assertion.

Comment In neither case is the “best possible” partial sum requested. Supporting evidence must be presented for the two partial sums given.

3. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n2^n}$ both converge (why?). By coincidence it turns out that their sums are both equal to $\ln 2$. (You'll understand this coincidence when we study Taylor series.)

Which series converges “faster” (and so numerically gives a more efficient way to get a numerical approximation for $\ln 2$)? Justify your answer by computing how many terms of each series must be added up to approximate $\ln 2$ with maximum allowed error of 10^{-6}.

4. Consider the following sequences:

$$a_n = \left(1 + \frac{1}{n}\right)^n \quad ; \quad b_n = \left(1 + \frac{1}{n^2}\right)^n \quad ; \quad c_n = \left(1 + \frac{1}{\sqrt{n}}\right)^n .$$

a) Use your calculator to plot the first ten terms of each of these sequences. Then use this information to guess the limiting behavior of each of the sequences.

b) Replace n by x and use L'Hopital's Rule to find the limit of each as x tends to infinity.

One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield's Math 152 webpage for this semester to learn which problem to hand in.