Continuity & differentiability

1. Here \( f(x) = \begin{cases} 
  x + 3 & \text{if } x \leq -2 \\
  \frac{1}{2}x^2 + A & \text{if } -2 < x
\end{cases} \) where \( A \) is a constant to be determined. Find \( A \) so that \( f(x) \) is continuous for all values of \( x \). Sketch a graph of \( y = f(x) \) using that value of \( A \) for \(-4 \leq x \leq 2\). Is \( f(x) \) differentiable at \( x = -2 \) using that value of \( A \)?

2. Here \( f(x) = \begin{cases} 
  Ax^2 - 1 & \text{if } x < -1 \\
  x + B & \text{if } -1 \leq x \leq 1 \\
  2 & \text{if } 1 < x
\end{cases} \) Find numbers \( A \) and \( B \) so that \( f(x) \) is continuous for all values of \( x \). Sketch a graph of \( y = f(x) \) for \(-3 \leq x \leq 3\).

3. In this problem \( f(x) = \begin{cases} 
  1 + x^2 & \text{if } x < 2 \\
  A + Bx & \text{if } -2 \leq x < 1 \\
  x^2 & \text{if } x \geq 1
\end{cases} \) Find \( A \) and \( B \) so that \( f(x) \) is continuous at all points. Sketch a graph of \( y = f(x) \) for \(-3 \leq x \leq 3\). For which values of \( x \) is \( f(x) \) not differentiable?

4. In the graph of \( y = f(x) \) to the right, identify with \( \text{m} \) any point which is a relative minimum; \( \text{M} \) any point which is a relative maximum; \( \text{C} \) any point which is a critical point; \( \text{I} \) any point which is an inflection point; \( \text{NC} \) any point at which \( f(x) \) is not continuous; and \( \text{ND} \) any point at which \( f(x) \) is not differentiable. Some points may have more than one label.

LETOVERS

Log/exp etc.

1. Find the range of \( f(x) = e^{-2x} + e^{3x} \).
2. Find the range of \( f(x) = \frac{\ln(x^2+1)}{x^2+1} \).