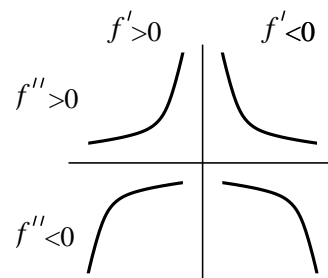


If  $\lim_{x \rightarrow b^-} f(x) = +\infty$  or  $\lim_{x \rightarrow b^-} f(x) = -\infty$  or  $\lim_{x \rightarrow b^+} f(x) = +\infty$  or  $\lim_{x \rightarrow b^+} f(x) = -\infty$ , then  $x = b$  is a **vertical asymptote** of the graph of  $y = f(x)$ .

If  $\lim_{x \rightarrow +\infty} f(x) = a$  or  $\lim_{x \rightarrow -\infty} f(x) = a$ , then  $y = a$  is a **horizontal asymptote** of the graph of  $y = f(x)$ .

### Shapes of curves

The signs of  $f'$  and  $f''$  determine {in|de}creasing and concave {up|down} behavior of the graph. All possibilities can occur: the signs of  $f'$  and  $f''$  can be independent. So pieces of  $y = f(x)$  can look like the curves to the right. The graphs of functions can bend *up* yet decrease. Functions can increase but also bend *down*. This may be weird but such behavior can occur.



Please try these problems. Expect some algebraic irritation. Some practice is good, though.

Q1. Suppose  $f(x) = \frac{x^2 + 3}{x^2 + x + 4}$ .

- What is the domain of  $f$ ? Find any horizontal or vertical asymptotes of  $f$ .
- Find any relative extrema of  $f$ . Find intervals where  $f$  increases and decreases.
- The *range* of a function is the collection of all possible values (outputs) of the function. What is the exact range of  $f(x)$ ? Explain your answer using calculus.

Q2. Suppose  $f(x) = \frac{e^x - 2}{e^x + 1}$ .

- What is the domain of  $f$ ? Find any horizontal or vertical asymptotes of  $f$ .
- Find any relative extrema of  $f$ . Find intervals where  $f$  increases and decreases.
- Find any inflection points of  $f$ . Find intervals where  $f$  is concave up and concave down.
- What is the exact range of  $f(x)$ ? Explain your answer using calculus.

Q3. Suppose  $f(x) = \frac{e^x + e^{2x}}{3e^x - e^{2x}}$ .

- What is the domain of  $f$ ? Find any horizontal or vertical asymptotes of  $f$ .
- Find any relative extrema of  $f$ . Find intervals where  $f$  increases and decreases.
- What is the exact range of  $f(x)$ ? Explain your answer using calculus.

Q4. Suppose  $f(x) = \frac{\sqrt{x^2 + 3}}{x + 1}$ .

- What is the domain of  $f$ ? Find any horizontal or vertical asymptotes of  $f$ .
- Find any relative extrema of  $f$ . Find intervals where  $f$  increases and decreases.
- What is the exact range of  $f(x)$ ? Explain your answer using calculus.