If \( \lim_{x \to b^-} f(x) = +\infty \) or \( \lim_{x \to b^+} f(x) = -\infty \) or \( \lim_{x \to b^-} f(x) = -\infty \) or \( \lim_{x \to b^+} f(x) = +\infty \), then \( x = b \) is a **vertical asymptote** of the graph of \( y=f(x) \).

If \( \lim_{x \to +\infty} f(x) = a \) or \( \lim_{x \to -\infty} f(x) = a \), then \( y = a \) is a **horizontal asymptote** of the graph of \( y = f(x) \).

**Shapes of curves**

The signs of \( f' \) and \( f'' \) determine \{increasing\}creasing and concave \{up\}\down\} behavior of the graph. All possibilities can occur: the signs of \( f' \) and \( f'' \) can be independent. So pieces of \( y = f(x) \) can look like the curves to the right. The graphs of functions can bend up\ yet decrease. Functions can increase but also bend down. \( f''<0 \)

This may be weird but such behavior can occur.

Please try these problems. Expect some algebraic irritation. Some practice is good, though.

**Q1.** Suppose \( f(x) = \frac{x^2 + 3}{x^2 + x + 4} \).

a) What is the domain of \( f \)? Find any horizontal or vertical asymptotes of \( f \).
b) Find any relative extrema of \( f \). Find intervals where \( f \) increases and decreases.
c) The **range** of a function is the collection of all possible values (outputs) of the function. What is the exact range of \( f(x) \)? Explain your answer using calculus.

**Q2.** Suppose \( f(x) = \frac{e^x - 2}{e^x + 1} \).

a) What is the domain of \( f \)? Find any horizontal or vertical asymptotes of \( f \).
b) Find any relative extrema of \( f \). Find intervals where \( f \) increases and decreases.
c) Find any inflection points of \( f \). Find intervals where \( f \) is concave up and concave down.
d) What is the exact range of \( f(x) \)? Explain your answer using calculus.

**Q3.** Suppose \( f(x) = \frac{e^x + e^{2x}}{3e^x - e^{2x}} \).

a) What is the domain of \( f \)? Find any horizontal or vertical asymptotes of \( f \).
b) Find any relative extrema of \( f \). Find intervals where \( f \) increases and decreases.
c) What is the exact range of \( f(x) \)? Explain your answer using calculus.

**Q4.** Suppose \( f(x) = \frac{\sqrt{x^2 + 3}}{x + 1} \).

a) What is the domain of \( f \)? Find any horizontal or vertical asymptotes of \( f \).
b) Find any relative extrema of \( f \). Find intervals where \( f \) increases and decreases.
c) What is the exact range of \( f(x) \)? Explain your answer using calculus.