(20) 1. Evaluate the indicated limits exactly. Give evidence to support your answers.

a) $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$

b) $\lim_{x \to -2^-} \frac{x^2 - x - 6}{|x + 2|}$

c) $\lim_{x \to 0} \frac{\sqrt{1 - x} - \sqrt{1 + x}}{3x}$

(12) 2. Suppose $f(x) = \dfrac{1}{x^2}$. Use the **definition of derivative** to find $f'(x)$.

(22) 3. Compute the derivatives of the following functions. Please do not simplify the answers.

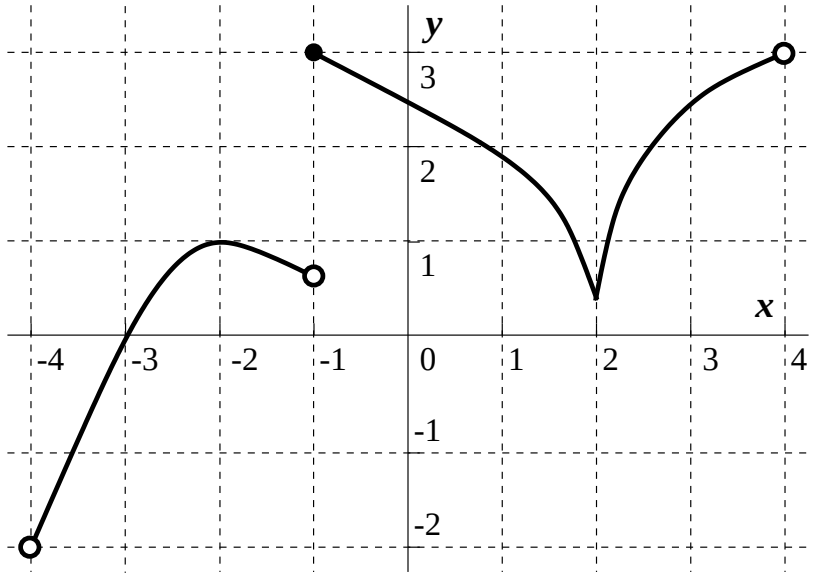
a) $\cos\left(x^4 + 3\right)$

b) $3e^{7x}(3x+5)^{1.4}$

c) $\dfrac{x^3+2}{5\ln x}$

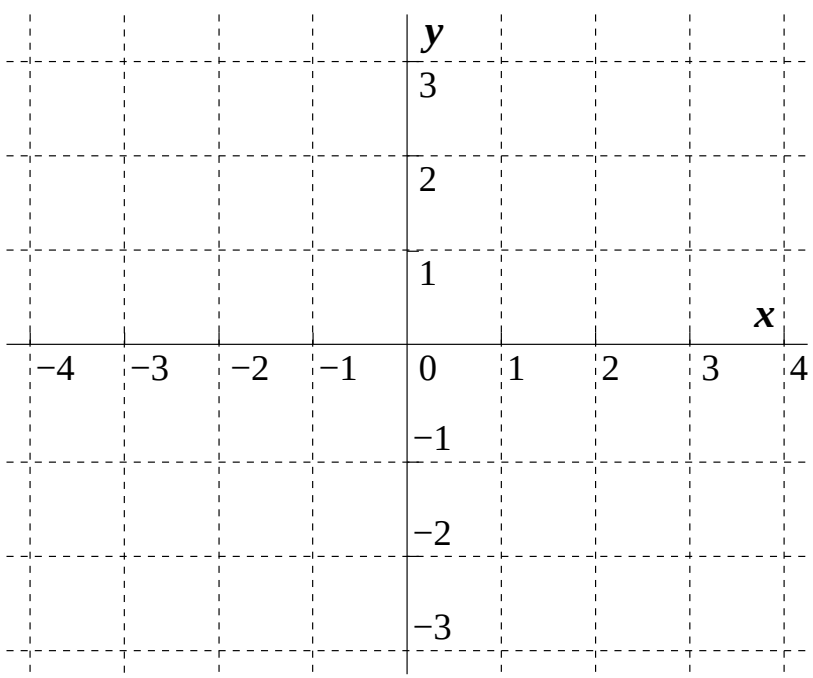
d) $\sqrt{\sin\left(\dfrac{1}{x^2}\right)}$

(12) 4. In this problem the function $f(x)$ has domain all x's between -4 and 4: $-4 < x < 4$. A graph of $y = f(x)$ is displayed below.



The graph of $y = f(x)$

Sketch a graph of $f'(x)$, the derivative of f, as well as you can on the axes below. Be sure to use the standard graphical abbreviations of this course: ● is a "filled dot", a point that *is* "there", on the graph; ○ indicates an "empty dot", a point that *isn't* "there".



The graph of $y = f'(x)$, the derivative of f

(12) 5. Suppose that $f(x)$ is a differentiable function, and that for all x, $4 < f'(x) < 6$. Suppose also that $f(0) = -2$.

a) Use a major theorem of this course to get some estimate of $f(5)$. That is, find numbers A and B so that $A \leq f(5) \leq B$. You should cite the result you are using explicitly and explain how the estimates you give follow from this result. Explain also why $f(5)$ must be a positive number.

b) Use a major theorem of this course together with the information given above and the conclusion of part a) to show that the equation $f(x) = 0$ must have at least one solution for x between 0 and 5. You should cite the result you are using explicitly and explain why this result implies the existence of the desired solution.

(18) 6. A rectangular box with a square base is to be made from two different materials. The material for the top and four sides costs \$1 per square foot, while the material for the bottom costs \$2 per square foot. If you can spend \$196 on materials, what dimensions will maximize the volume of the box?

Comment $196 = 14^2$.

(12) 7. Find equations for all horizontal and vertical asymptotes of

$$f(x) = \frac{x + 2\sqrt{x^2 + 3}}{4x + 5}\,.$$

(14) 8. Suppose that $f(x) = \sqrt{17 - x^3}$. Then $f(2) = 3$.

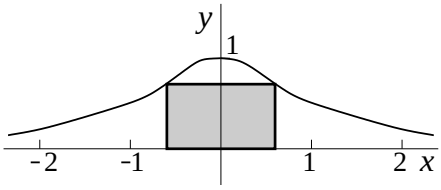
a) Use linear approximation to get an approximate value for $f(1.97)$. You do **not** need to simplify your answer!

b) 3.058533472 is the true value of $f(1.97)$ to ten-digit accuracy. Explain briefly using calculus (not calculator evidence!) how you could have predicted that the true value is likely to be greater than or less than the approximate value found in a).

(12) 9. a) Give an example of a function whose domain is all real numbers which is continuous everywhere *except* at one point. Explain why your example is not continuous at that point. If you give a "piecewise" example, be sure that the domain is all real numbers.

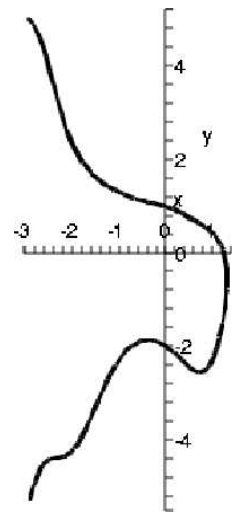
b) Give an example of a function whose domain is all real numbers which is differentiable everywhere *except* at one point. Explain why your example is not differentiable at that point. If you give a "piecewise" example, be sure that the domain is all real numbers.

(12) 10. A rectangle is bounded by the x-axis and the curve $y = \dfrac{1}{1+x^2}$ as shown in the figure to the right. One side of the rectangle is on the x-axis and the two opposite corners are on the curve. What length and width should the rectangle have so that its area is a maximum?



Briefly explain using calculus why your answer gives a maximum.

(12) 11. A graph of a portion of the curve described implicitly by the equation $y^2 + x^3 + y + y\sin(x + x^2) = 2$ is shown to the right.



a) Find the coordinates of the two points of intersection of this curve with the y-axis (where $x = 0$).

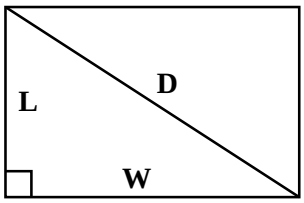
b) Are the tangent lines to $y^2 + x^3 + y + y\sin(x + x^2) = 2$ at the two points where the curve intersects the y-axis parallel? Use calculus to answer this question.

(16) 12. Suppose $f(x) = (x^2 - 3)e^x$.

a) Find the first coordinates (the x values) of all relative maxima and minima of the function $f(x) = (x^2 - 3)e^x$. Briefly explain your answers using calculus.

b) Find the first coordinates (the x values) of all inflection points of the function $f(x) = (x^2 - 3)e^x$. Briefly explain your answers using calculus.

(14) 13. At a certain time, a rectangle has **L**ength equal to 5 inches and **W**idth equal to 7 inches. Also at that time, the **L**ength is <u>increasing</u> at .3 inches per second and the **W**idth is <u>decreasing</u> at .4 inches per second.



a) Compute the area of the rectangle at the certain time. Is the area increasing or decreasing at that time, and at what rate? Put your answers in the spaces indicated.

Area = _____ inches^2

Rate of change of area = _____ inches^2 per second

The area is ____creasing (use one of {in|de}).

b) Compute the length of the diagonal of the rectangle at the certain time. Is this length increasing or decreasing at that time, and at what rate? Put your answers in the spaces indicated.

Diagonal's length = _____ inches

Rate of change of diagonal's length = _____ inches per second

The diagonal's length is ____creasing (use one of {in|de}).

(12) 14. In this problem, $f(x) = 3x + \dfrac{12}{x}$.

Find the absolute minimum and absolute maximum values of f in the interval $\left[\frac{1}{2}, 4\right]$.

Final Exam for Summer Bridge: Foundations of Calculus

August 2, 2013

NAME _____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators or other electronic devices may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	20	
2	12	
3	22	
4	12	
5	12	
6	18	
7	12	
8	14	
9	12	
10	12	
11	12	
12	16	
13	14	
14	12	
Total Points Earned:		