Please do all problems.

1. Suppose that $u$ and $v$ are the real and imaginary parts respectively of an entire function $f$. Find all $f$ such that $u = v^2$.

2. Suppose $U$ is an open subset of $\mathbb{C}$. Show that there is a sequence of compact subsets of $U$, $\{K_n\}_{n \in \mathbb{N}}$ so that $\bigcup_{n=1}^{\infty} K_n = U$ with each $K_n$ contained in the interior of $K_{n+1}$.

3. Suppose that $f$ is an entire function, and that for all $|z| > 7$, $|f(z)| \leq 5e^{(|z|^2)}$. Find explicit positive numbers $A$ and $B$ so that if $|z| > A$, then $|f''(z)| \leq Be^{(3|z|^2)}$

4. Suppose $f(z) = \frac{1}{z^2(e^z - 1)}$.
   a) Find and classify (removable, pole, essential) all isolated singularities of $f$. If the isolated singularity is a pole, tell the order of the pole and the residue of $f$ at the pole.
   b) Compute $\int_{\sigma} f(z) dz$ where $\sigma$ is the closed curve displayed to the right.

5. What is the automorphism group of $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$? That is, describe all biholomorphic mappings from $\mathbb{C}^*$ to itself. Decide whether or not this group is transitive. If it is not, describe the orbit and stabilizer of 1, and also describe the orbit and stabilizer of 2.

   **Note** The orbit of a point under a collection of mappings is the set of image points. The stabilizer (also called the stabilizer subgroup or the isotropy subgroup) of a group of bijections is the subgroup which keeps the indicated object (here, a point) fixed.

6. Suppose $f$ is an entire function, and that, for all $z \in \mathbb{C}$, $f(z + i) = f(z + 1) = f(z)$ (so $f$ is doubly periodic with periods $i$ and 1). Prove that $f$ is constant.