That is ~~thus~~ is $\pi : \mathbb{C}^{n+1 *} \to \mathbb{CP}^n$. $\mathbb{C}^{n+1 *}$ has a nice topology. On $\mathbb{CP}^n$ put the smallest topology making π continuous: $U \subset \mathbb{CP}^n$ is open $\Leftrightarrow \pi^{-1}(U)$ is open in $\mathbb{CP}^n$. Heuristically, a sequence (ζ_n) in $\mathbb{CP}^n$ converges when? When there are representatives $z_n \in \pi^{-1}(\zeta_n)$ so that (z_n) converges to z in $\mathbb{C}^{n+1 *}$ (not to 0, note — we can always get representatives of equivalence classes to converge to 0!).

In fact both $\mathbb{CP}^n$ and $\mathbb{RP}^n$ are compact. Why is this? Let $\|z\|^2 = \sum_{j=0}^{n} |z_j|^2$. Then the map

$\mathbb{C}^{n+1 *} \xrightarrow{Q} S^{2n+1}$ given by $Q(z) = \left(\frac{z_0}{\|z\|}, \dots, \frac{z_n}{\|z\|}\right)$ is

$\mathbb{R}^{n+1 *} \xrightarrow{Q} S^n$

continuous, and is "coarser" than the quotient map, π. That is, if $Q(z) = Q(\tilde{z})$, then $z \sim \tilde{z}$.

$$\begin{tikzcd} \mathbb{C}^{n+1 *} \arrow[r, "Q"] \arrow[dr, "\pi"'] & S^{2n+1} \arrow[d, dashed, "?"] \\ & \mathbb{CP}^n \end{tikzcd} \qquad \begin{tikzcd} \mathbb{R}^{n+1 *} \arrow[r, "Q"] \arrow[dr, "\pi"'] & S^{n} \arrow[d, "?"] \\ & \mathbb{RP}^n \end{tikzcd}$$

In fact, the "fiber" of "?" in both cases is easy to determine.

For if two unit vectors in the sphere represent the same equivalence class, then, since they have the same length, $|\lambda| = 1$. In the real case, the fiber of ? has 2 elements (because of ± 1). In the complex case, the fiber is a circle.