a circle/line if P is real, $R = \overline{Q}$, and S is real.
(sufficient, not necessary!)
Consider $Pz\bar{z} + Rz + \bar{R}\bar{z} + S = 0$. Change $z \to \mathcal{D}_B z$

Then we get: $\underbrace{PB\bar{B}}_{\text{real}}\, z\bar{z} + \underbrace{RB}_{}z + \underbrace{\bar{R}\bar{B}}_{}\bar{z} + \underbrace{S}_{\text{real}} = 0$

conjugate of each!

Change $z \to \tau_A z$. Then we get

$$P(z+A)(\bar{z}+\bar{A}) + R(z+A) + \bar{R}(\bar{z}+\bar{A}) + S =$$

$$\underbrace{P}_{\text{given real}} z\bar{z} + (P\bar{A} + R)z + (PA + \bar{R})\bar{z} + \underbrace{(S + PA\bar{A} + RA + \bar{R}\bar{A})}_{\text{and this is real}}$$

so these are conjugate.

and this is real ($w \in \mathbb{R} \Leftrightarrow \bar{w} = w$!)

Change $z \mapsto \mathcal{I}z = \frac{1}{z}$:

$$P\frac{1}{z}\frac{1}{\bar{z}} + R\frac{1}{z} + \bar{R}\frac{1}{\bar{z}} + S = 0 \text{ becomes:}$$

$$P + R\bar{z} + \bar{R}z + Sz\bar{z} = 0.$$

P: real; $R\bar{z}$, $\bar{R}z$: still conjugate of one another; S: real

So we (almost!) done. One remark before going on, though: $\mathcal{I}z = \frac{1}{z} = \frac{\bar{z}}{|z|^2}$. What is this geometrically? It is pretty: