$$\frac{a}{c} + \frac{\left(\frac{b}{c} - \frac{ad}{c^2}\right)}{z + \frac{d}{c}} \underset{\text{Whew!}}{=} \mathcal{T}_{\frac{a}{c}}\left(\mathcal{D}_{\frac{b}{c}-\frac{da}{c^2}}\left(\mathcal{I}\left(\mathcal{T}_{\frac{d}{c}}(z)\right)\right)\right).$$

Wow!

So to prove that $LF(\mathbb{C})$ leaves the family of lines and circles invariant involves merely convincing you that $\mathcal{T}_A$, $\mathcal{D}_B$ and $\mathcal{I}$ do so.

What are "lines and circles"? They are the "loci" of these real equations: $p(x^2+y^2) + qx + ry + s = 0$ (here p, q, r, s are real, and we consider the set of all (x, y) satisfying these equations. In complex "language",

$x^2 + y^2 = z\bar{z}$, $x = \frac{1}{2}(z+\bar{z})$, $y = \frac{1}{2i}(z - \bar{z})$, so

$p(x^2+y^2) + qx + ry + s$ becomes $(p)(z\bar{z}) + \frac{q}{2}(z+\bar{z}) + \frac{r}{2i}(z-\bar{z}) + s =$

$p(z\bar{z}) + \left(\frac{q - ir}{2}\right)z + \left(\frac{q+ir}{2}\right)\bar{z} + s.$

Conversely, if we consider those z's so that z & $\bar{z}$ satisfy $Pz\bar{z} + Qz + R\bar{z} + S = 0$, then this "comes from"