

In fact, even more is true. $LF(\mathbb{C})$ doesn't know the difference between lines and circles. (Look at how \mathbb{E} and \mathbb{H} appear in $\mathbb{C}P^1$, if you don't believe me).

"Prop:" $LF(\mathbb{C})$ transforms $\{\text{lines, circles}\} \subseteq \mathbb{C}P^1$ into itself. (In fact, it acts transitively on them.)

Proof: We'll need some strategy to prove this (as I see it). I want to think about both of the complicated objects in the proposition.

First, $LF(\mathbb{C})$. I claim $LF(\mathbb{C})$ is generated (as a group) by translation ($T_A(z) = z + A$), dilation ($D_B(z) = Bz$, with $B \neq 0$) and the inversion $I(z) = \frac{1}{z}$. How can we see this? Consider

$$M(z) = \frac{az+b}{cz+d}. \quad \text{If } c=0, M(z) = \frac{a}{d}z + \frac{b}{d} = T_{\frac{b}{d}} \circ D_{\frac{a}{d}}(z).$$

$$\text{If } c \neq 0, \text{ consider } \frac{az+b}{cz+d} = \frac{\frac{a}{c}z + \frac{b}{c}}{z + \frac{d}{c}} = \frac{\frac{a}{c}(z + \frac{d}{c}) - \frac{ad}{c^2} + \frac{b}{c}}{z + \frac{d}{c}} =$$