Math 421  Some vibration examples  November 29

I defined a small triangular initial condition for Maple:

```maple
> F:= x -> piecewise(x<Pi/3,0,x<Pi/3+Pi/12,x-(Pi/3),x<Pi/2,Pi/3+Pi/6-x,0);
```

This was an initial perturbation of the string. Here is a picture of the initial perturbation, together with the sum of the first 10 terms of its Fourier sine series. To the right is a similar picture, except that what’s shown is the sum of the first 100 terms of its Fourier sine series. I can’t see any difference between the two curves in the picture on the right.

The math equations in back of this: \( b_n = \frac{2}{\pi} \int_0^\pi F(x) \sin(nx) \, dx \), so that the partial sum of the Fourier sine series is \( Q_N(x) = \sum_{n=1}^N b_n \sin(nx) \).

Now let’s “solve” the wave equation with this initial data, and with the boundary conditions corresponding to the ends fastened at 0 and \( \pi \): so we want \( u(x,t) \) satisfying: **PDE** \( u_{xx} = u_{tt} \); **BC** \( u(0,t) = 0; u(\pi,t) = 0 \) for all \( t \); **IC** \( u(x,0) = F(x) \) and \( u_t(x,0) = 0 \), both for \( 0 \leq x \leq \pi \).

The approximate solution will be \( V_N(x) = \sum_{n=1}^N b_n \sin(nx) \cos(nt) \). Here are pictures for various \( t \)’s:

\[
\begin{align*}
  &t = 0.2 & t = \frac{\pi}{6} & t = \frac{\pi}{3} \\
  &t = \frac{\pi}{2} & t = \frac{2\pi}{3} & t = \pi
\end{align*}
\]
Now I'd like to solve an initial velocity problem. Here I'll suppose that the initial velocity of the string is up one unit in the interval $[\frac{\pi}{6}, \frac{\pi}{2}]$.

> \text{G:} x \rightarrow \text{piecewise}(x < \pi/3, 0, x < \pi/2, 1, 0);

And here is a picture of the Fourier sine series, first for $n = 10$ and then for $n = 100$:

![Graph 1](image1)

The math equations in back of this: $c_n = \frac{2}{\pi} \int_{\pi}^{\pi} G(x) \sin(nx) \, dx$, so that the partial sum of the Fourier sine series is $Q_N(x) = \sum_{n=1}^{N} c_n \sin(nx)$.

Now let's "solve" the wave equation with this initial data, and with the boundary conditions corresponding to the ends fastened at 0 and $\pi$: so we want $u(x,t)$ satisfying: \( \text{PDE} \ u_{xx} = u_{tt} ; \ \text{BC} \ u(0,t) = 0; u(\pi,t) = 0 \) for all $t$; \( \text{IC} \ u(x,0) = 0 \) and $u_x(x,0) = G(x)$, both for $0 \leq x \leq \pi$.

The approximate solution will be $V_N(x) = \sum_{n=1}^{N} \frac{A_n}{\pi} \sin(nx) \sin(nt)$. Here are pictures for various $t$'s:

![Graph 2](image2)

$t = .1$

$t = .4$

$t = 1.2$

$t = 1.7$

$t = 2.1$

$t = 3$