(12) 1. Complete the definitions.

a) Suppose v_1, v_2, ... and v_t are vectors in $\mathbb{R}^n$. Then a *linear combination* of v_1, v_2, ... and v_t is

b) Suppose A is an n by n matrix. λ is an *eigenvalue* of A if

(18) 2. Suppose that $A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}$.

a) Compute the characteristic polynomial of A.

b) Find a basis of $\mathbb{R}^2$ consisting of eigenvectors of A.

c) Find a diagonal matrix D and an invertible matrix P so that $P^{-1}AP = D$.

d) Find P^{-1}.

e) Compute $Z = AP$.

f) Compute $P^{-1}Z$.

(12) 3. For which values of x is the matrix $A = \begin{pmatrix} 1 & 0 & 1 & 1 \\ x^2 & 3 & 2 & 4 \\ x & 4 & 2 & 3 \\ 1 & 1 & 1 & 0 \end{pmatrix}$ invertible?

(14) 4. Suppose the vectors $(1,0,3,2,-2)$ and $(-1,1,2,2,5)$ are solutions to a homogeneous system, $AX = 0$, where A is a 5 by 5 matrix, to be used in all parts of this problem.

a) Are there other solutions to this homogeneous system? What can be concluded about the dimension of the collection of all solutions, S, of this homogeneous system?

b) Estimate the rank of A.

c) Consider the inhomogeneous system, $AX = Y$. Does this system have solutions for all Y's in $\mathbb{R}^5$?

(16) 5. The matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ has eigenvalues 1 and 2 and 3, with associated eigenvectors $(-1,1,0)$ and $(0,0,1)$ and $(1,1,0)$. Use this information together with diagonalization to compute A^5.

Note $1^5 = 1$ and $2^5 = 32$ and $3^5 = 243$. The entries in the answer are 0, 32, 121, or 122.

(16) 6. **BIRD** Suppose $\mathbf{u} = (2,3,1,5,1)$ and $\mathbf{v} = (0,5,3,1,-1)$ and $\mathbf{w} = (2,6,-2,-2,5)$.

a) Show that $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly independent in $\mathbb{R}^5$.

b) Find a vector in $\mathbb{R}^5$ which is *not* a linear combination of $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$. Verify your answer.

(12) 7. **BIRD** Use linear algebra to find a polynomial $p(x)$ of degree 3 so that $p(1) = 2$, $p'(1) = -2$, $p(2) = 2$, and $p'(2) = -2$.

Second Exam for Math 421, section 2

April 8, 2004

NAME ____

Do all problems, in any order.
Show your work. An answer alone may not receive full credit.
No notes other than the distributed formula sheet may be used on this exam.
No calculators may be used on this exam.

Problem Number	Possible Points	Points Earned:
1	12	
2	18	
3	12	
4	14	
5	16	
6	16	
7	12	
Total Points Earned:		

New Jersey native birds
some matrices and their reduced row echelon forms

GOLDFINCH (3 by 6) $\left(\begin{array}{ccccc|c} 2 & 3 & 1 & 5 & 1 & a \\ 0 & 5 & 3 & 1 & -1 & b \\ 2 & 6 & -2 & -2 & 5 & c \end{array}\right) \sim \left(\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{17}{6} & \frac{5}{12} & \frac{7}{12}a - \frac{1}{4}b - \frac{1}{12}c \\ 0 & 1 & 0 & -\frac{3}{4} & \frac{3}{8} & -\frac{1}{8}a + \frac{1}{8}b + \frac{1}{8}c \\ 0 & 0 & 1 & \frac{19}{12} & -\frac{23}{24} & \frac{5}{24}a + \frac{1}{8}b - \frac{5}{24}c \end{array}\right)$

NUTHATCH (5 by 4) $\left(\begin{array}{ccc|c} 2 & 0 & 2 & a \\ 3 & 5 & 6 & b \\ 1 & 3 & -2 & c \\ 5 & 1 & -2 & d \\ 1 & -1 & 5 & e \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & \frac{7}{12}a - \frac{1}{8}b + \frac{5}{24}c \\ 0 & 1 & 0 & -\frac{1}{4}a + \frac{1}{8}b + \frac{1}{8}c \\ 0 & 0 & 1 & \frac{1}{12}a + \frac{1}{8}b - \frac{5}{24}c \\ 0 & 0 & 0 & -\frac{17}{6}a + \frac{3}{4}b - \frac{19}{12}c + d \\ 0 & 0 & 0 & -\frac{5}{12}a - \frac{3}{8}b + \frac{23}{24}c + e \end{array}\right)$

HELICOPTER (4 by 5) $\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & a \\ 0 & 1 & 2 & 3 & b \\ 1 & 2 & 4 & 8 & c \\ 0 & 1 & 4 & 12 & d \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4a - 4b + 5c - 2d \\ 0 & 1 & 0 & 0 & 12a + 8b - 12c + 5d \\ 0 & 0 & 1 & 0 & -9a - 5b + 9c - 4d \\ 0 & 0 & 0 & 1 & 2a + b - 2c + d \end{array}\right)$

MOSQUITO (4 by 5) $\left(\begin{array}{cccc|c} 1 & 0 & 1 & 0 & a \\ 1 & 1 & 2 & 1 & b \\ 1 & 2 & 4 & 4 & c \\ 1 & 3 & 8 & 12 & d \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -4a + 12b - 9c + 2d \\ 0 & 1 & 0 & 0 & -4a + 8b + 5c + d \\ 0 & 0 & 1 & 0 & 5a - 12b + 9c - 2d \\ 0 & 0 & 0 & 1 & -2a + 5b - 4c + d \end{array}\right)$