Due at the beginning of class, Thursday, October 9, 2008

These problems finish up our consideration of basic topology. On to analysis!
I decided that I couldn’t push you folks through some of the intricate problems such as 23 through 26, which are more part of a topology course like Math 441. I give you one problem which might be useful to know, and five more textbook problems.

(10 points) Suppose $X$ is a set with two metrics, $d$ and $D$. Suppose that the following logical statements are true (in what follows, $N^d_r(x)$ and $N^D_r(x)$ are, respectively, the open balls of radius $r > 0$ around $x \in X$ in the $d$, respectively, $D$ metric):

If $x \in X$ and $t > 0$, then there is $s > 0$ so that $N^d_s(x) \subset N^D_t(x)$.

If $x \in X$ and $t > 0$, then there is $s > 0$ so that $N^D_s(x) \subset N^d_t(x)$.

Prove that a subset of $X$ is open using $D$ if and only if it is open using $d$.

Comment This can be done with great economy if you think a bit.

Textbook problems Chapter 2: 16, 17, 20, 22, 24, and 29. All are worth 10 points.