1. Suppose \( f(x) = (1 - x)^{-1/2} = \frac{1}{\sqrt{1-x}} \).

a) Find the fourth Taylor polynomial, \( T_4(x) \), centered at \( a = 0 \) for \( f \).

b) Sketch the graphs of \( y = f(x) \) and \( y = T_4(x) \) in the window \([-1, 1] \times [0, 3]\).

c) Sketch the graph of \( f(x) - T_4(x) \) in the window \([-0.5, 5] \times [-0.01, 0.01]\).

d) Use Taylor’s inequality (the Error Bound) to find an overestimate for \(|f(x) - T_4(x)|\) on the interval \([-0.5, 0.5]\). This should be an explicit number valid for all \( x \)'s in this interval.

2. The horizontal and vertical axes on this graph have different scales. \( x \) goes from \(-10\) to \(10\) and \( y \) goes from \(-1\) to \(3.5\). The graph is a direction field for the differential equation \( y' = \frac{1}{10} \left(1 - \frac{1}{10} y x^2\right) \).

a) Sketch the solution curve which passes through \((0, 1)\) on the graph.

b) How many critical points does this solution curve seem to have? What types of critical points do they seem to be? If \((x_0, y_0)\) is a critical point, find an exact algebraic relationship between \(x_0\) and \(y_0\).

**Comment** The equation can’t be solved in terms of standard functions. Information from the graph and the differential equation should be used.

3. Consider the differential equations

\[ \begin{align*}
\text{a) } \frac{dy}{dx} &= 2x + 3y \\
\text{b) } \frac{dy}{dx} &= e^{2x+3y} \\
\text{c) } \frac{dy}{dx} &= x^3 y^2 \\
\text{d) } \frac{dy}{dx} &= x^2 + y^3
\end{align*} \]

Two of these are separable. For each of these two separable equations, solve the initial value problem with the initial condition \( y(0) = 1 \). In each case your solution should be written as \( y = f(x) \) where \( f(x) \) is a formula. Choose one of the non-separable equations and explain carefully why it is not separable.

4. A 200-gal tank contains 100 gal of water with a salt concentration of 0.1 lb/gal. Water with a salt concentration of 0.4 lb/gal flows in the tank at a rate of 20 gal/min. The fluid is mixed instantaneously, and water is pumped out at a rate of 10 gal/min. Let \( y(t) \) be the amount of salt in the tank at time \( t \).

a) Set up and solve the differential equation for \( y(t) \).

b) What is the salt concentration when the tank overflows?

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This is a problem from the textbook: #28 in section 9.5.

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One problem will be selected for a writeup to be handed in at the next recitation meeting. Please see Professor Greenfield’s Math 152 webpage to learn which problem to hand in.